7^{th} International Conference & Exposition on Petroleum Geophysics

"HYDERABAD 2008"

P - 311

Eaton's Equation and Compaction - A study

K.M. Sundaram*, Rita Jain
Oil & Natural Gas Corporation Ltd, Mumbai, email: sndrmkm@yahoo.co.in

Summary

Eaton's Equation says that if there are two levels having same over burden, one normally compacted and the other not, if subscript n stands for normal and Φ_{ze} for Effective stress, then

$(\Phi_{ze})_{obs} = (\Phi_{ze})_n \quad (Y_{obs}/Y_n)^k$ where Y is a property such as resistivity, 'obs' stands for 'observed' and k is an exponent. …… (Ref. 1)

This approach, as well as the Equivalent Depth Method of TrangOH (Ref. 2) assume that Effective stress and Porosity are uniquely related. Thereby pore pressure increase over normal-compaction-case pore pressure results in lowering of Effective stress for a given overburden. And therefore, that this effective stress change is reflected in Porosity change wrt normally compressed sediments' porosity for same overburden.

Let us assume that the property Y is Resistivity. Rw being same for normally compacted and observed cases for same overburden, the above equation would then reduce to :

$(\Phi_{ze})_{obs} = (\Phi_{ze})_n \,.\, Ø_n^{1.8k}/Ø_{obs}^{1.8k}$ assuming Archies Equation with Archie cementation factor to be 1.8.

From this it is seen that constancy of $(\Phi_{ze}) \,.\, Ø^{1.8k}$ is a necessary and sufficient condition for Eaton's Equation.

This paper examines the applicability of the above condition from the perspective of the Compaction Process

Introduction

When changes in porosity in response to change in stress is analyzed and a porosity effective stress relationship arrived at,it is possible to have certain insight into Eaton equation that are worth future exploration. The present paper through the work flow presented examines these aspects.

Theory and/or Method

Changes in Porosity in response to change in stress:

Changes in Bulk Porosity of saturated Rock to changes in Vertical stress are identical to changes in Porosity of frame to changes in Effective Stress.

In the following, $Ø_e$ stands for Porosity excluding adsorbed water on clay surfaces.

Suppose Porosity at Effective stress Φe is $Ø_e$
And let $Øe+\Delta Øe$ denote Porosity when Effective stress is changed from Φe to Q_e+Q_e

Let $k_Ø$ denote the Pore Elastic modulus

Then, by definition, $\Delta Ø_e = - Ø_e \, \Delta Ø_e / k_Ø$ ---- (i)

Now if K stands for Compression modulus of Elasticity

$1/K_{frame} = 1/K_{grain} + Ø / k_Ø$ ---- (ii) (pl. see Appendix for proof)

From Equations (i) and (ii) above we have

$\Delta Øe = - \{1/K_{frame} - 1/K_{grain}\} \; \Delta\Phi e$

$\Delta Øe = - 1/K_{grain} \{ K_{grain} / K_{frame} - 1\} \; \Delta\Phi e$ ---- (iii)

$K_{frame} = AK_{grain} (1- Øe)^c$ ….. (iv) where C is an exponent (pl. see Appendix 5 and also Ref. 3) where A is a constant and a measure of grain to grain contact elasticity. A has been taken equal to 1.0 in the present study.

Thus $\{K_{grain} / K_{frame} - 1\} = \{(1- Ø_e)^{-C}-1\} \approx cØ_e$

Substituting in (iii) we get $\Delta\Phi e = -(C\,Øe)/ K_{grain} \cdot \Delta\Phi e$ ---- (v)

(The Equation (iv) can be written as $C_{frame} - C_{grain} (1-Øe)^{-c}$
which is similar to Archies Equation $R_o = R_w \cdot Ø_e^{-m}$

The 'Pore Volume' of Archie is substituted by 'Grain volume' c, the equivalent of Archie cementation factor can be expected to vary with porosity).

When rock is more and more compacted, Φ dependance of c on Øe becomes weaker and weaker and disappears after a critical porosity is crossed, or (as is Limestones) grain to grain cementation is complete, and reaches its extreme value.

Porosity – Effective Stress Relationship :

We hypothesise that $C = Ø_e^{+a}$, where a is a +ve number greater than 1

Then $CØe = Ø_e^{(1+a)}$ ---- (vi)

Substituting $C\,Ø_e$ from Equation (vi) above in Equation (v), we get

$\Delta Ø_e = - Ø_e^{(1+a)} / K_{grain} \cdot \Delta Ø_e$

In terms of differentials we can write

$Ø_e^{-(1+a)}\, d\,\Phi^e = -1/K_{grain} \cdot d\,\Phi^e$

Integrating both sides we get

$(Ø_e)^{-a} / -a = -1/K_{grain} \cdot \Phi^e + C_1$

or $(Ø_e)^{-a} = a / K_{grain}\ \Phi e + C_1$

At mud line Φe = 0. Let Øe at mud line be Øo

Substituting in above, we get $C1 = Øo^{-a}$

i.e $\Phi e = K_{grain} / a \ [1/ Ø_e^{a} - 1/ Ø_o^{a}]$ ……...(vii)

we shall now examine 2 subcases of the above equation.

Subcase (a) - when $Ø_e$ is << $Ø_o$:

In this sub case $1/ Ø_e^a >> 1/ Ø_o^a$ Consequently

$Ø_e$ can be approximated to :

$Ø_e \approx K_{grain} / a \ . \ 1/(Øe)^a$ ……(viii)

Assuming Biot's Poro Elastic constant $\alpha \approx 1$

$Øe = P_{ovb} - P_{pore}$

Consider 2 levels having same overburden, one part of normally compacted sediments sequences and the other the level under observation.

For the first case $(\Phi_e)_n = P_{ovb} - (P_{pore})$ …. (ix)

For the second case $(\Phi_e)_{obs} = P_{ovb} - (P_{pore})_{obs}$ ….. (x)
ie observation case

Equ (x) can be written as $(P_{pore})_{obs} = P_{ovb} - (\Phi_e)_{obs} = P_{ovb} - [(\Phi)_{obs}/(\Phi_e)_n].(\Phi_e)_n$

or ie $(P_{pore})_{obs} = P_{ovb} - [P_{ovb} - (P_{pore})_n] \cdot [\Phi e_{obs} / \Phi e_n]$ …….. (xi)

Using Equ. (viii), if $(Ø_e)_{obs}$ and $(Ø_e)_n$ stand for Porosity values for observation case and normally compacted case,

Than $(\Phi_e)_n = K_{grain} / a \ . (1/(Ø_e)_n^{a})$

$(\Phi_e)_{obs} = K_{grain} / a \ . (1/(Øe)_{obs}^{a})$

Let R_{obs} be resistivity for observation case and R_n be that for normally compacted case.

Then $R_{obs} / R_n = [(Ø_e)_n]^m / [(Ø_e)_{obs}]^m$ where m is Archie cementation Exponent.

$(R_{obs} / R_n)^y = (\Phi_e)_{obs} / (\Phi_e)_n$ …….. (xii),
(where y = a/m

Substituting from (xii) into (xi) for $(\Phi_e)_{obs} / (\Phi_e)_n$, we get

$(P_{pore})_{obs} = P_{ovb} - [P_{ovb} - (P_{pore})_n] \cdot [R_{obs} / R_n]^y$ ….. (xiii),
where y = q/m

Equation (xiii) is nothing but Eaton Equation (Ref. 1)

Thus we can conclude that :

(a) Applicability of Eqn. (viii), with

(b) 'a', 'K_{grain}' unchanging with overburden (ie Depth)
is a necessary and sufficient condition for Eaton Equation.

(c) We also note that physical meaning of Y is that it is a/m. 'm' can be taken to be 1.8 with abundant reasonability. 'a' can be determined by computing a from Eqn (viii) {if they are sufficiently deep, else from Eqn. (xiv)} from different normally compacted levels and arriving at a reasonable measure of central tendency, statistically, or forward modeled by finding 'c' at different levels using log (1- $Ø_e$ vs Log K_{frame} plots (Ref. 3)

(d) Thus a 'correctly' framed Eaton Equation can be used because Y is now 'determined' frp, actual data. since y = a/1.8 with value of 'a' found as discussed at (c) above.

(e) Eaton Equation is a special case only, when depths are not too near mud line.

Sub-case (b) - when $Ø_e$ and $Ø_e$ are comparable

In this subcase we no longer can say $1/Øe^a >> 1/Øo^a$

Consequently $\Phi_e = K_{grain} / a \ [1/(Ø_e)^a - 1/(Ø_o)^a]$ ………..(xiv)

$(\Phi_e)_{obs} = K_{grain} / a \ [1/(Øe)_{obs}^a - 1/Øo^a] = K_{grain}/a \ [(R_{obs})^y - (R_{mudline})^y]$ ……(xv)

and

$((\Phi_e)_n = K_{grain} /a \ [1/(Ø_e)_n^a - 1/(Ø_o)^a] = K_{grain} / a \ [(R_{nor})^y - (R_{mudline})^y]$ …xvi
where y = a/m
We would again have Eqn (Xa)

$(P_{pore})_{obs} = P_{ovb} - (\Phi_e)_{obs} = P_{ovb} - \{(\Phi_e)_{obs} / (\Phi_e)_n) . ((\Phi_e)_n) \}$

or ie $(P_{pore})_{obs} = P_{ovb} - [P_{ovb} - (P_{pore})_n\} \ [(\Phi_e)_{obs} / (\Phi_e)_n]$

Substituting into Equation above, from Eqns. Xv, xvi, we get

$(P_{pore})_{obs} = P_{ovb} - [P_{ovb} - (P_{pore})_n] . \ [R_{obs}^y - R_{mudline}^y] / [R_{nor}^y - R_{mudline}^y]$ …….(xvii)

Equation (xvii) can be considered as a Generalised Eaton Equation.

The Derivation of Eqn (xiii) analogue and Eqn (xvii) analogue for Sonic Compressional Travel time case is illustrated at Appendix 3.

Application of the work discussed above to Porosity Decline :

The foregoing work leads to Porosity Decline Equation $D1 = \{1/(\Delta_{av} . \Delta_w)_g\} \ (K_{grain} / a) \ . \ (1/Øe^a)$ - (Appendix 2 may please be referred to for a deriviation of this equation)

Here D_1 is depth for Mud line, Δ_{av} is average Sediment Density, Δ_w is average water density considered.

Appendix 1

Relation between K_{frame}, $Ø_e$, $K_Ø$, K_{grain}

For simplicity we can consider unit volume of frame.

A stress change of $\Delta\Phi_e$ on frame causes Bula volume change in frame of $\Delta\Phi_e / K_{frame}$

A stress change of $\Delta\Phi_e$ on frame is equivalent to a stress change of $\Delta\Phi_e / (1-Ø_e)$ on frame scaffolding.

Consequently volume change in frame scaffolding = $\Delta\Phi_e/(1-Ø_e) . (1/K_{grain}) . (1-Ø_e)$

Change in Pore volume $(\Delta Ø_e)$ is therefore equal to
$\Delta Ø_e = \Delta\Phi_e/K_{frame} - \Delta\Phi_e/K_{grain}$ …….(i)

By definition $K_Ø = \Delta\Phi_e$ / { Change in Pore volume / $Ø_e$ }

Change in Pore volume $(\Delta\Phi_e) = \Delta\Phi_e . 1/K_o . Ø_e$

Equating (i) and (ii) we get

$1/K_{frame} - 1/K_{grain} = Ø_e / K_Ø$

or ie $1/K_{frame} = 1/K_{grain} + Ø_e/K_Ø$

From the above, we note that $1/K_Ø = [1/K_{frame} - 1/K_{grain}] \ 1/Ø_e$

ie $1/K_Ø = 1/K_{grain} . Ø_e \ [K_{grain} / K_{frame} - 1]$

K_{frame} and K_{grain} can be related through $K_{frame} = K_{grain} (1-Ø_e)^c$

Where c can have a variance with $Ø_e$

We would then have $1/K_Ø = 1/K_{grain} . Ø_e . [(1-Ø_e)^{-c} – 1]$

$= c/K_{grain}$ to a fair approximation.

Or $K_Ø = K_{grain} / c$ where c can vary with $Ø_e$, thus $K_Ø$ varies with $Ø_c$

Appendix 2

Implications to Porosity Decline with Depth in case of Normal Compaction

We start with Equation (vii) namely

$Φ_e = K_{grain} / a\ [1/Ø_e^a - 1/Ø_o^a]$ …………..(vii)

$Φ_e = Φ_{ovb} - (Δ_{pore})_n$ …… (xviii)

where $(Δ_{pore})_n$ = Pore Pressure under conditions of normal compaction. If $Δ_{av}$ is average sediment density upto Point of observation which is representative.

$Φ_{ovb} = gΔ_{av} (D-D_m)$ (where D_m is Mud line depth) + $D_m\ Δ_w g$ where $Δ_w$ is Average water density.

$Φ_{ovb} = Δ_{av}\ (D-D_m)g + D_m . Δ_w g$ where g is Accelaration due to Gravity.

Substituting in Eqn (vii) we get, using Eqn. (xviii)

$Δ_{av}\ (D-D_m)g + D_m . Δ_w g - (Δ_{pore})_n = K_{grain} / a\ [1/Ø_e^a – 1/Ø_o^a]$

ie $D. Δ_{av} . g = g (Δ_{av} – Δ_w) D_m + (Δ_{pore})n + K_{grain} / a\ [1/Ø_e^a – 1/Ø_o^a]$ …...xix

$(Δ_{pore})_n = Δ_w . D.g$

Substituting for $(Δ_{pore})n$ from above, we get

$(D-D_m)\ Δ_{av}\ g = (D – D_m)\ Δ_w g + K_{grain} / a\ [1/Ø_e^a - 1/Ø_o^a]$

or ie. $D-D_m)\ (Δ_{av} - Δ_w)\ g = K_{grain} / a\ [1/Ø_e^a - 1/Ø_o^a]$

If $D1 = D-D_m$ = Depth below Mud line,

$D1 = \{ 1/(Δ_{av} – Δ_w)g\} \{K_{grain} / a\} . 1/Ø_e^a$, approx. $1/Ø_o^a = 1$,

which is reasonable ……………….. (x)

To add

This is the Decline Equation, that the preceeding arguments lead to. The salient feature of the arguments developed in this paper is that KØ is a function of Porosity

Recalling that $dΦe/doe = K_Ø . 1/Ø_e$, and that $dΦ = Pav.dD.g$,

the assumption of $K_Ø$ being a constant would have lead to

$ØD / d_{Øe}\ (Δ_{av} / K_Ø) = 1/Ø_e$ which in turn would have

led to conventional decline equation $Ø_e = A_e$ -8 D where $8 = [P_{av} /K_Ø]$

The form of equation (x) above is clearly different from the above form. It is also different from the form $(D-D_m) = .(10^{-bØe})$

Where b and . are constants. (Ref. 5)

Appendix 3

Analogue of Eqn (xiii) and Eqn (xvii) when Sonic Interval Velocity is the Property chosen to reflect compaction. From the standard equations for sonic interval transit time, it can be seen that when Matrix acoustic velocity for compressional body wave >> bulk compressional body wave velocity denoted V,

$<^2 /Ø_e$ where X is a constant (pl. see Appendix 4) ------(A)

Consequently we would have $(Φ_e)_n = (K_{grain} / a)\ [(<_n^2/X)^a]$ ----(B)

$(Φ_e)_{obs} = K_{grain} / a . [V^2_{obs}/X]^a$ where 'n' stands for normal compaction case and 'obs' for observation case.

Then it is easily seen that

$(Δ_{pore})_{obs} = Δ_{ovb} – [Δ_{ovb} – (Δ_{pore})_n] . (V_n / V_{obs})^y$ where Y = 2a

Similarly $(Δ_{pore})_{obs} = Δ_{ovb} – [Δ_{ovb} – (Δ_{pore})_n]\ [V_n^y – V_{mudline}^y / V_{obs}^y – V_{mudline}^y]$ where y = 2a

The value of y can be estimated, from V, $Φ_e$ correlations. For example is S Kagens method (Ref.6)

$V_n = M\ Φ_{en}^{1/3}$ where M is a constant which implies $Φ_{en} /Φ_{obs} =(V_n/V_{obs})^3$, and which therefore indicates value of y to be 3.0

Appendix 4

According to Gassman Equation

$K = K_{frame} + [\{1-(K_{frame} / K_{grain})2\} / \{ Ø_e / K_{fluid} + 1\text{-}O / K_{grain} – K_{frame} / K_{grain}{}^2\}]$

$K/K_{grain} = K_{frame} / K_{grain} + \{(1/K_{grain} – K_{frame} / K_{grain}{}^2) (1\text{-} K_{frame} / K_{grain})] / \{ Ø_e /K_{fluid} + 1\text{-}Ø / K_{grain} – K_{frame} / K_{grain}{}^2\}$

Assuming $K_{frame} / K_{grain} <<1$, and thereby also neglecting $K_{frame} / K_{grain}{}^2$ in above, we get

$K / K_{grain} = (K_{frame} / K_{grain}) + (1/K_{grain} . 1/ (Ø_e /K_{frame} 1\text{-} Ø_e /K_{grain})) = \{1/K_{grain} [1/(Ø_e /K_{fluid} + 1\text{-} Ø_e /K_{grain})] \} + (1\text{-}Ø_e)^c$

Assumption of $Ø_e$ to be sufficiently large, that

$(1\text{-}Ø_e)^c$ is << the quantity in braces, leads to K/K_{grain} to be approximately to $(1/K_{grain}) / \{ Ø_e /K_{fluid} + (1\text{-} Ø_e) / K_{grain} \}$

Under these conditions K becomes

$K = 1/\{ Ø_e /K_{fluid} + (1\text{-} Ø_e) / K_{grain}\}$ approx.

Or $1/K = Ø_e /K_{fluid} + (1\text{-} Ø_e)/K_{grain}$

Under circumstances such as above, which are true when $Ø_e$ exceeds around 37% G_{frame} where G stands for shear modules, and which is equal to G the bulk shear modules, will be low compared to K.

Consequently $1/\Delta V2$ approximates to $Ø_e /K_{fluid} + 1\text{-} Ø_e /K_{grain}$

Where $l\Delta$ is bulk density of rock.

Since $K_{grain} >> K_{fluid}$, and where $Ø_e$ is large enough, $(1\text{-} Ø_e)$ is comparable to $Ø_e$, $Ø_e / K_{fluid} >> 1\text{-} Ø_e /K_{grain}$ and we can approximate

$1/\Delta V^2$ to $Ø_e /K_{fluid}$, ie $1/\Delta V^2 = Ø_e /K_{fluid}$ approx $= Ø_e /\Delta_{fluid} V_{fluid}{}^2$

ie $V^2 = \Delta_{fluid} / \Delta . V_{fluid} Ø_e$

$\Delta_{fluid} / \Delta = \Delta_{fluid} / Ø_e (\Delta_{fluid} – \Delta_{ma}) + \Delta_{ma}$. $Ø_e$ being in Denominator, where $Ø_e$ is sufficiently high (Δ_{fl} / Δ has weak variation with $Ø_e$ and so $V^2 = A$ constant $X . Ø_e$ where $X = V_{fluid} / [0.4 (\Delta fl –\Delta_{ma}) + \Delta_{ma})$ approx.

Appendix 5

It is known from empirical data that Vp /Vs for dry-gas bearing clean sandstones is a constant and thus independent of Porosity (Ref. 4). It is also known that G_{frame} and Porosity $Ø_e$ are related by $G_{frame} = G_{grain} (1\text{-} Ø_e)^c$ where c is an exponent -------(a)
w
From the above two results, it was deduced that for porous clean sands Kframe also shall be related to $Ø_e$ through $K_{frame} = AK_{grain} (1\text{-} Ø_e)^c$, since independence of (V_p/V_s) of dry gas (approximating to frame) from $Ø_e$ implies independence of $(V_p/V_s)^2$ from $Ø_e$ which in turn implies $(Kframe + 4/3\, G_{frame}) / G_{frame}$ to be independent of $Ø_e$ and thereby K_{frame} / G_{frame} to be independent of $Ø_e$.

This is possible only if K_{frame} and K_{grain} are related as $K_{frame} = Ak_{grain}, (1 - Ø_e)^c$ …(b) where A is a constant.

This constant A is a measure of Grain + Grain contact Elasticity. Similarly G grain in Eqn (b) also needs to be replaced by BG grain where B is a measure of grain and grain Contact Elasticity.

$G_{frame} = B_{grain} (1\text{-} Ø_e)^c$ …………(c)

The validity of (b) and (c) for porosity ranges which occur is clean sandstone reservoir has been seen (ref. 3) and hence the Eqns. have been adopted.

A, B have been taken to be equal to 1.0 respectively in this work.

Conclusions:

1. Sediment Porosity being sufficiently low as compared to mudline porosity ie levels not too near mudline is a precondition for Eaton Equation to be valid.

2. Eaton Equation's Exponent has a physical meaning and can be evaluated from normal compaction trends or forward modeled for best fit - achieved, of forward model of K_{frame} vs Log $(1\text{-} Ø_e)$ with actual Log K_{frame} vs Log $(1\text{-} Ø_e)$ plots.
3. The assumption that 'c' the exponent in $K_{frame} = K_{grain} (1\text{-} Ø_e)^c$, has a variation with Ø via $a = Ø^a$ is a valid assumption, since it is a necessary condition for Eaton's Equation, (which is a valid popularly and Eqn) to be valid.
4. It is demonstrated that in addition to modeling the 'y' Exponent Eaton Equation, Eaton Equation itself can be generalized to Eqn of form of Eqn (xvii).
5. Generated Eaton Eqn (Eqn. Xvii) as suggested in this paper and ways of estimation of y, as

suggestd in this paper have potential for usability and usefulness to have a Compaction Equation, which can be used to forward Model Log Responses for normal compaction case.

6. Usefulness of Approach to Porosity Decline Eqn Studies (ie Compaction studies is demonstrated, which enables forward modeled compaction trends to be generated which help in editing out less than acceptable data quality points, not uncommon as hole sizes are large and porosity is high.

Acknowledgement :

The author is indebted to ONGC and specially to ED-COED, WOB for having kindly encouraged to carry out this study. The views expressed are solely of the authors.

References:

1. Eaton B.A 1975 The equation for GeoPressure prediction from Ell logs SPE 50th Annual Fall Meeting Proceedings SPE 5544.
2. Trugott M. O 1997 Pore/Fracture Pressure Determination in Deep Water: Deep Water Technology 218(8), Supplement to World Oil pp 68-70.
3. Brie A Pampuri F, Mrsala A F , Meazza O Shear Sonic Interpretation in gas bearing ssands SPE 30595 1995, 701-710.
4. Dhruba J Dutta K M Sundaram Compressional and shear velocity data dentify and quantify fluids in carbonates and clastics SPWLA symposium 2004.
5. Rasmus JC Forward Modeling of log response in geopressured formation reveals valuable insights to the various pore pressure prediction techniques AAPG Memoirs 76 pp 159-164.