Comparison of local time-frequency decomposition and continuous wavelet transform for seismic spectral decomposition

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Summary

Spectral decomposition techniques are widely used in oil industry for detection of frequency anomalous features in seismic data. In last few years many advanced techniques are developed. Local time-frequency decomposition (LTFD) is recently developed spectral decomposition technique which uses regularized nonstationary regression with Fourier bases. In the present paper, we compare LTFD with CWT in terms of time and frequency resolution capabilities. Chirp signals, Berlage wavelets and Ricker wavelets of different frequencies are used for performing comparison.

Introduction

Seismic spectral decomposition refers to mathematical techniques that map seismic data from time domain to time vs frequency domain. In time vs frequency domain, frequency anomalous features are apparent which are otherwise difficult to observe in seismic data. Spectral decomposition is widely used in oil industry for interpretation of various frequency anomalous features such as direct hydrocarbon indicator (DHI) (Castagna et al., 2003; Sinha et al., 2005), high-frequency anomalies in carbonate reservoir (Li and Zheng, 2008; Li et al., 2011) etc.

Development of spectral decomposition technique with high time and frequency resolution is an active area of research for a long time. Different spectral decomposition techniques have been developed in past few years. Basic spectral decomposition technique is short time Fourier transformation (STFT) (Gabor, 1946) which suffers from the time-frequency resolution limitation due to fixed window size (Chakraborty and Okaya, 1995). Continuous wavelet transform (CWT) (Grossmann and Morlet, 1984) is a popular spectral decomposition technique and uses variable length windows in contrast to STFT where fixed length window are used. Recently Liu and Fomel (2013) developed local time-frequency decomposition (LTFD) which uses regularized nonstationary regression with Fourier bases. They used it for ground-roll noise attenuation and multicomponent image registration.

In the present paper, we compare LTFD and CWT techniques qualitatively for seismic spectral decomposition. For this purpose, spectral decomposition of different chirp signals (with and without noise) and synthetic seismic wavelets (Ricker and Berlage wavelets) is performed and resultant spectrograms are compared.

Theory

Local time-frequency decomposition

LTFD is a Fourier transform based technique in which signal is decomposed into Fourier components by using regularized nonstationary regression (Liu and Fomel, 2013).
Fourier series of input signal $f(t)$ on $[0, L]$ can be expressed as

$$f(t) = a_0 + \sum_{k=1}^{\infty}[a_k \cos\left(\frac{2k\pi t}{L}\right) + b_k \sin(\frac{2k\pi t}{L})] \quad (1)$$

where a_k and b_k are the series coefficients and k is related to frequency by $k=Lf$.

Equation (3) can be written as

$$f(t) = \sum_{k=0}^{\infty} c_k \Psi_k(t) \quad (2)$$

where $c_k = [a_k \quad b_k]$ and $\Psi_k(t) = \begin{bmatrix}\Psi_{1k}(t)\\ \Psi_{2k}(t)\end{bmatrix} = \begin{bmatrix}\cos\left(\frac{2k\pi t}{L}\right)\\ \sin\left(\frac{2k\pi t}{L}\right)\end{bmatrix}$

In linear notation, time varying coefficients $c_k(t)$ can be obtained by solving following least-squares problem,

$$\min_{c_k(t)} \sum_{k=0}^{N} ||\, f(t) - c_k\Psi_k(t)||_2^2 \quad (3)$$

where $||\ \ ||_2^2$ denotes the squared L_2 norm of a function. Equation (5) represents ill-posed problem and for solving it, regularization term is added.

$$\min_{c_k(t)} \sum_{k=0}^{N} ||\, f(t) - c_k\Psi_k(t)||_2^2 + R[c_k(t)] \quad (4)$$

where R is a regularization operator.

The coefficients $c_k(t)$ are estimated using shaping regularization (Fomel, 2007) which is similar to problem of nonstationary regression. After estimating coefficients $c_k(t)$, time-frequency map is calculated as

$$F(t, f = k/L) = \sqrt{a_k^2(t) + b_k^2(t)} \quad (5)$$

For more details on LTFD, interested readers may refer to Liu and Fomel (2013).

Continuous wavelet transform

CWT is a widely used mathematical tool for time-frequency analysis of a non-stationary signal. CWT is defined as the inner product of mother wavelet with the input signal f (t) and is given by

$$W(a, \tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t)\Psi^*\left(\frac{t-\tau}{a}\right)dt \quad (6)$$

where W are CWT coefficients, Ψ^* is complex conjugate of the mother wavelet which is shifted in time 'τ' and stretched by a scale 'a'. Mother wavelet should be selected judiciously by observing input signal and optimal mother wavelet is one whose features matches with features of input signal. The scale parameter 'a' can be transformed to pseudo-frequency using the equation (2) (Mallat and zhang, 1993; Reine et al., 2009),

$$F_a = \frac{F_c}{a\,.\,\Delta} \quad (7)$$

For in-depth discussion of CWT, interested readers may refer to Daubechies (1992) and Mallat (2008).

Results of comparative study

We compare LTFD and CWT using various numerical signals.

Chirp signal

In chirp signal, frequency increases (up-chirp) or decreases (down-chirp) with time either linearly (linear chirp) or exponentially (exponential chirp). The exponential chirp can be generated using following equation.

$$f(t) = f_0 k^t \quad (8)$$

where f_0 is frequency at $t=0$ sec and k is the rate of frequency.

In order to test performance of CWT and LTFD, we have generated different exponential chirp signals as shown in Figure 1. The range of frequency of these different exponential chirp signals is from 1 – 100 Hz. Spectral decomposition using CWT and LTFD is performed and resultant spectrograms are shown in Figure 1.

It is seen that both the techniques can resolve two chirps in all signals. Comparison of spectrograms for two exponential chirps with constant amplitude (Figure 1.a) indicates that CWT incorrectly shows decrease in maximum amplitude for higher frequencies (more than 70 Hz) while LTFD shows uniform amplitude for lower and higher frequencies. Similarly spectrograms for two exponential chirps with linearly decreasing amplitudes (Figure 1.c) indicates that CWT incorrectly shows minimal amplitudes of signal between 80 – 100 Hz while LTFD shows average amplitudes between 80 – 100 Hz.

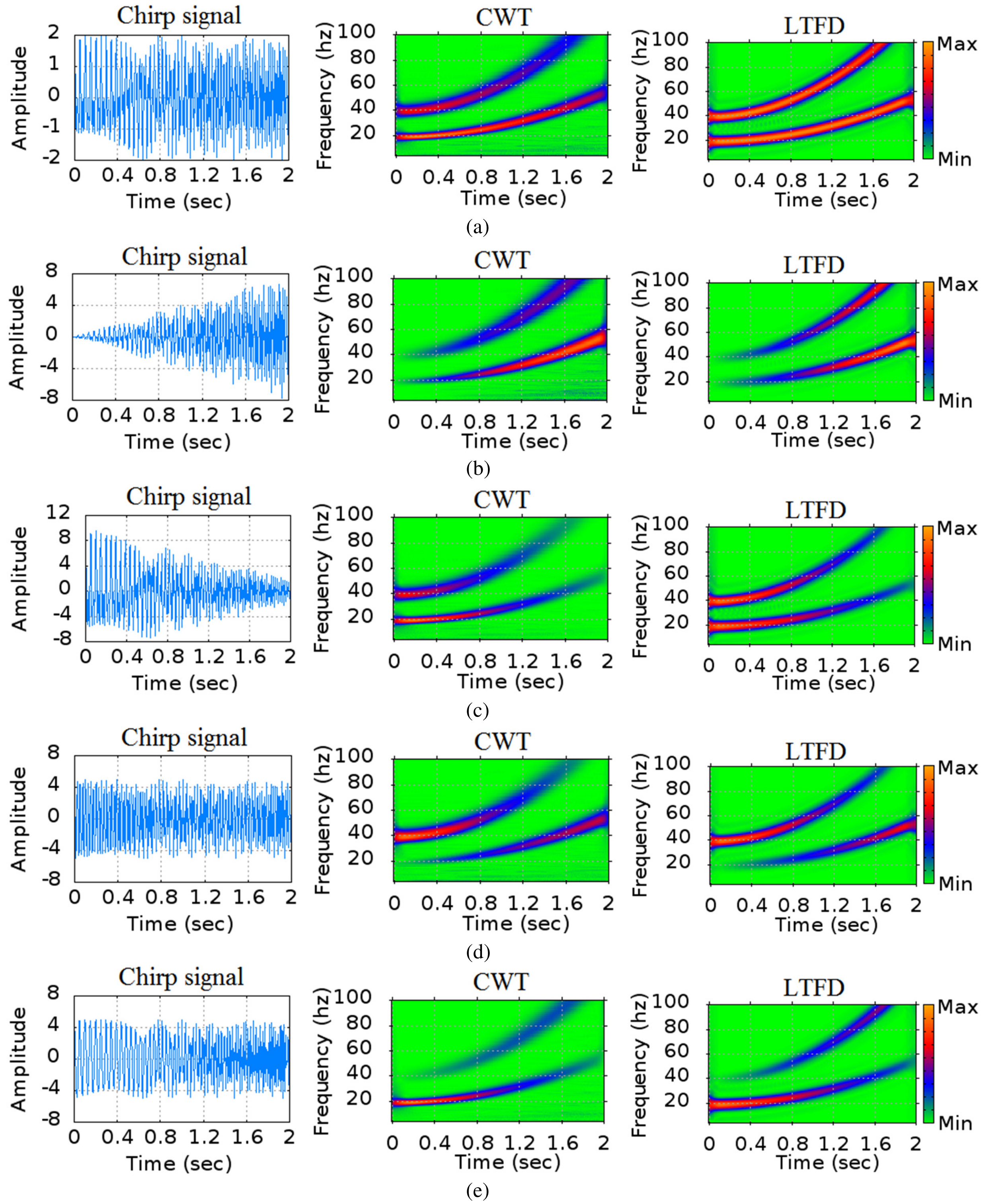

Figure 1: CWT and LTFD spectrograms of synthetic signal consisting of two exponential chirp signals with (a) constant amplitude (b) linearly increasing amplitudes (c) linearly decreasing amplitudes (d) amplitude of 1st chirp increasing and 2nd chirp decreasing (e) amplitude 1st chirp decreasing and 2nd chirp increasing.

Berlage wavelet

The Berlage wavelet (Aldridge, 1990) is defined by the equation

$$w(t) = AH(t)t^{n}e^{-\propto t}\cos(2\pi f_0 t + \varphi_0) \qquad (9)$$

where $H(t)$ is the Heaviside unit step function [$H(t) = 0$ for $t \leq 0$ and $H(t) = 1$ for $t > 0$], $\propto$ is a nonnegative exponential decay factor, n is a nonnegative time exponent, f_0 is the peak frequency and φ_0 is a phase angle. The Berlage wavelet is generally used for modeling marine seismic data where Berlage wavelet mimics airgun source signature (Symons et al., 2006).

Berlage wavelets of different peak frequencies are generated and analyzed with CWT and LTFD and resultant spectrograms are compared (Figure 3). Berlage wavelets have zero amplitude before $t = 0$ sec. LTFD spectrogram of Berlage wavelets with peak frequency of 10 Hz and 30 Hz do not show energy before t = 0 sec while CWT spectrogram shows some energy before t = 0 sec which is erroneous.

FFT spectrum of Berlage wavelet of peak frequency of 10 Hz indicates that signal contains frequencies upto 80 Hz only. The same information is evident in LTFD spectrogram while CWT spectrogram erroneously shows frequencies till 100 Hz. FFT spectrum of Berlage wavelet of peak frequency of 50 Hz indicates that signal contains frequencies below 20 Hz. The

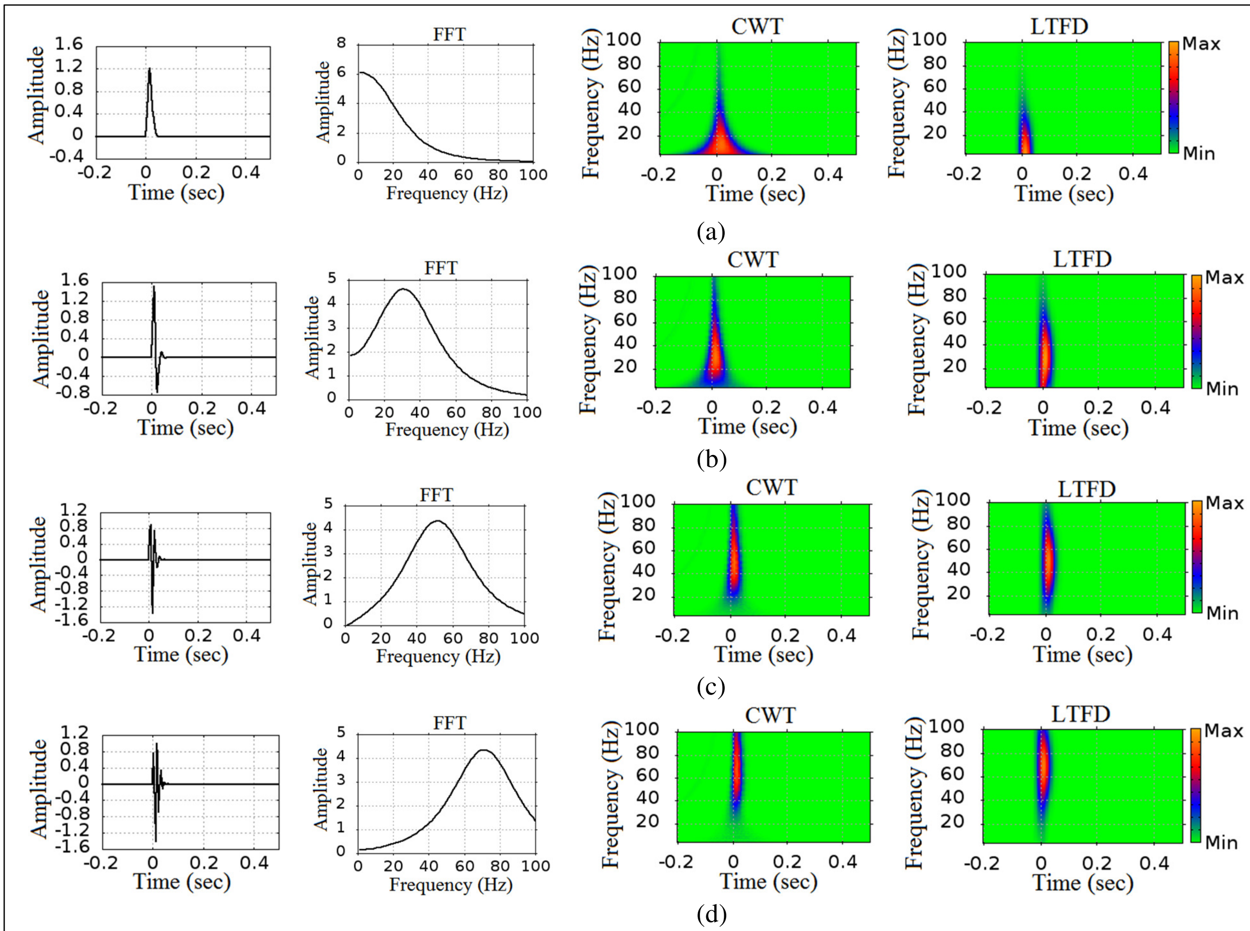

Figure 3: FFT spectrum, CWT spectrogram and LTFD spectrogram of Berlage wavelet of (a) 10 Hz peak frequency (b) 30 Hz peak frequency (c) 50 Hz peak frequency (d) 70 Hz peak frequency.

same information is evident in LTFD spectrogram while CWT spectrogram erroneously shows very minimal or no frequency below 20 Hz.

Ricker wavelet

Ricker wavelet (Ricker, 1940) is defined by the equation

$$A = \left(1 - 2\pi^2 {f_p}^2 t^2\right) e^{-\pi^2 {f_p}^2 t^2} \qquad (10)$$

where f_p is the peak frequency. Ricker wavelet is widely used as source wavelet in modeling and other techniques. Ricker wavelets of different peak frequencies are generated and are analyzed with CWT and LTFD and resultant spectrograms are compared (Figure 4). FFT spectrum is also provided for reference (Figure 4).

Ricker wavelet in time domain consists of one peak and two troughs and it is symmetric around peak. FFT spectrum of Ricker wavelet consists of one peak and it is approximately symmetric around peak. Ideally spectrogram of Ricker wavelet should show symmetry around maximum amplitude along time and frequency. Such symmetry is apparent on LTFD spectrograms. In CWT spectrograms, signal is broader at lower frequencies and narrower at higher frequencies which is poor spectrum of Ricker wavelet. FFT spectrum of Ricker wavelet of peak frequency of 30 Hz indicates that signal contains frequencies upto 80 Hz only. The same information is evident in LTFD

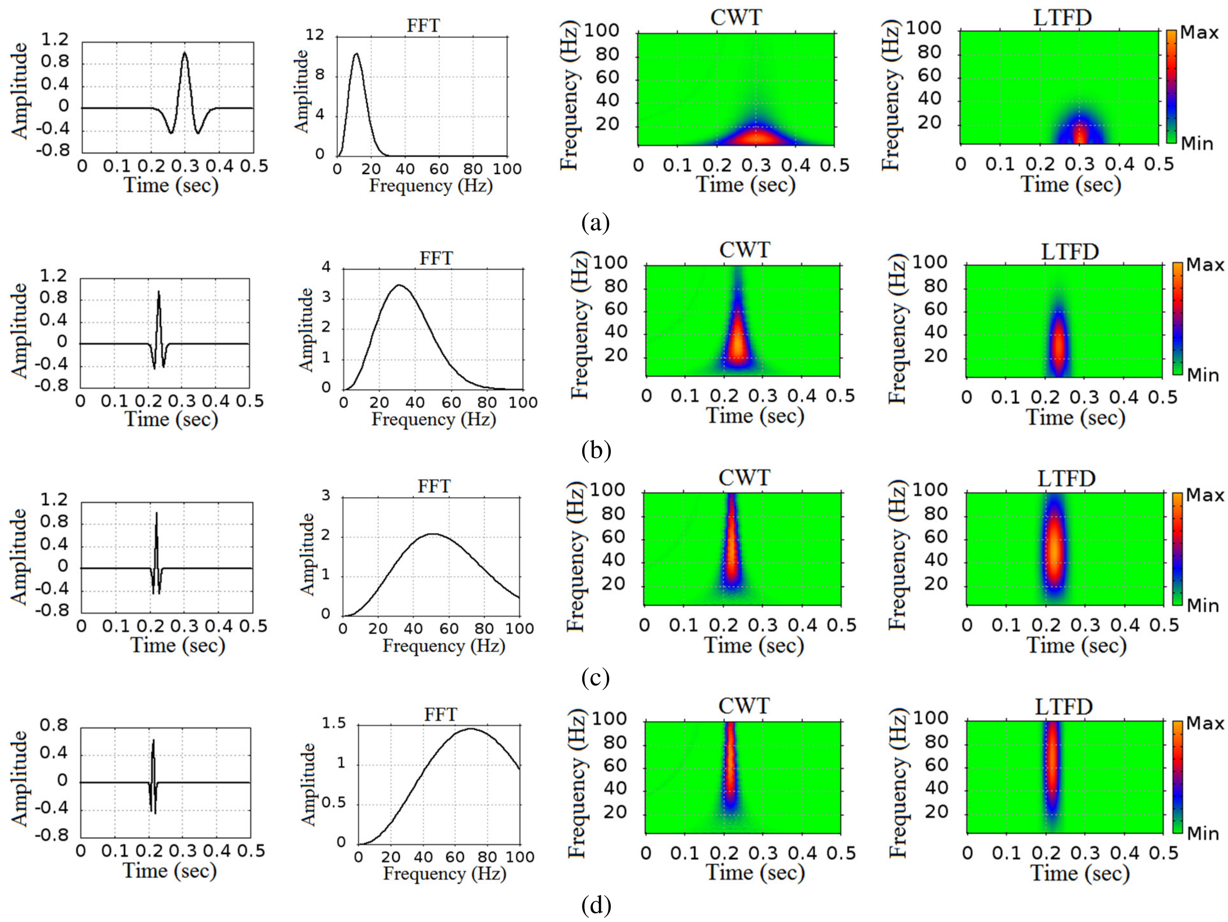

Figure 4: FFT spectrum, CWT spectrogram and LTFD spectrogram of Ricker wavelet of (a) 10 Hz peak frequency (b) 30 Hz peak frequency (c) 50 Hz peak frequency (d) 70 Hz peak frequency.

spectrogram while CWT spectrogram erroneously shows frequencies till 100 Hz.

Conclusions

CWT is a popular spectral decomposition technique while LTFD is a recently developed technique. Qualitative comparison of LTFD and CWT in terms of time and frequency resolution is carried out using chirp signals, Ricker and Berlage wavelets of different frequencies. In light of the experiments performed, LTFD seems promising over conventional CWT in terms of temporal and spectral resolution. The applicability of the LTFD for detection of low frequency shadows for hydrocarbon interpretation is a matter of exploration in near future.

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