Sensitivity of effective pressure in carbonates using Biot's and experimentally determined effective stress coefficient.

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Summary
Interdependence of effective pressure (P_e) and acoustic velocities (compressional V_p and shear V_s) in granular media is often exploited as one of the main rock physical attribute for reservoir characterization. This is particularly relevant for 4D feasibility investigations experiencing simultaneous pressure and saturation changes. Effective pressure changes in carbonate reservoir, however could be trickier because of the change in effective stress coefficient (α) as a result of rock and fluid interaction. Here, we calculated changes in effective pressure in a carbonate reservoir and its overburden shales based on commonly practiced; Terzhagi (1923), the Biot's and Willis (1957) using Biot's coefficient (β), and the experimentally determined effective stress coefficient (α) explained by Todd and Simmons (1972). We found that the calculated effective pressure was markedly different for the three methods, where P_e using α exhibit values between the two extremes represented by other two methods. We consider this information crucial because it exhibits that correlations between acoustic velocities and effective pressure could change drastically leading to very different interpretations of the changes due to pressure and saturation. It also signifies that experimentally determined effective stress coefficient is crucial to prediction of effective pressure in carbonate reservoirs that generally do not follow compaction trends.

Introduction
Effective pressure prediction and its reliance on pore pressure and effective stress coefficient plays a crucial role in successful exploration and exploitation of hydrocarbons. Predicting the changing effective stress condition is critical for avoiding drilling problems and non-productive operational time, inside the reservoirs it can also influence mechanical and petrophysical properties and therefore the subtle seismic signatures responsible for successful reservoir characterization (Sharma et al. 2013). Effective stress actually governs the seismic signatures through seismic velocities (Ebrom and Heppard 2003 & 2007, Vasqeuz et al. 2009). Whereas Terzhagi (1923) is mostly for the unconsolidated sediments and gives out differential pressure (P_d) as P_e, Biot's and Willies (1957) using Biot's coefficient (β) (equation 1 &2), has been the mainstay for quite some time for calculating effective pressure under hydrostatic conditions.

Effective Pressure Calculation
$$P_e = P_d = P_c - P_p \quad \text{.......... (1)}$$

$$P_e = P_c - \beta P_p \quad \text{.......... (2)}$$

where,

$$\beta = 1 - \frac{K_{dry}}{K_{min}} \quad \text{.......... (3)}$$

where, K_{dry} is the dry rock modulus obtained using the rearranged Gassmann relation (equation 4) given by Zhu and McMechan (1990).

$$K_{dry} = \frac{K_{sat}\left(\frac{\varphi K_{min}}{K_{fl}}+1-\varphi\right)-K_{min}}{\frac{\varphi K_{min}}{K_{fl}}+\frac{K_{sat}}{K_{min}}-1-\varphi} \quad \text{.......... (4)}$$

Where,

$$K_{fl} = \frac{1}{\left(\frac{V_w}{K_w}+\frac{1-V_w}{K_{hc}}\right)} \quad \text{.......... (5)}$$

$$K_{sat} = \rho_b * \left(V_p^2 - \frac{4}{3} * V_s^2\right) \quad \text{.......... (6)}$$

$$K_{min} = \frac{1}{2}\left((V_{sh}K_{sh} + V_{non-sh}K_{nonsh}) + \frac{1}{\left(\frac{V_{sh}}{K_{sh}}+\frac{V_{non-sh}}{K_{non-sh}}\right)}\right) \quad \text{..(7)}$$

Other input parameters (equation 5 to 7) in Equation 4 were obtained from logs and literature. In absence of exact Vclay estimates, volume of shale (V_{sh}) itself was considered as a lithology unit and calcite as the non-shale lithology. The pore fluid was considered as brine with bulk modulus 2.4 GPa. Parametric details are shown in Table 1. For simplicity sake, it was assumed that saturation was uniform although it is rarely the case in carbonates particularly under production behavior (Cadoret 1992).
Not only do we have assumptions such as shale bulk modulus (K_{sh}) and fluid bulk modulus (K_{fl}) while calculating K_{dry} above, elastic constants for mineral bulk modulus (K_{min}) calculation also remained constant irrespective of the sample depth. This can pose significant differences in prediction of effective pressure using Biot's coefficient calculated as

above. Not to miss the point that in carbonates reservoirs (other than chalk types) logs do not show compaction dependence which is typical of clastic setting.

Table 1. Elastic constants and Δt values for matrix and fluid

Mineral/ Fluid	Bulk Modulus (GPa)	Δt (µs/m)
Calcite	75	155
Shale	21	205
Brine	2.4	616

To overcome the above limitations in Biot's coefficient, we used experimentally determined effective stress coefficients (α) given by Todd and Simmons (1972).

$$\alpha = 1 - \frac{(dQ/dP_p)_{P_d}}{(dQ/dP_d)_{P_p}} \qquad (5)$$

Where, Q is any measured physical property. Figure 1 presents the schematics behavior of compression-wave velocity with respect to confining pressure. The two derived quantity for constant differential and constant pore pressure are indicated. Substituting 'α' in place of 'β' in equation 2, we estimate 3rd variant of effective pressure.

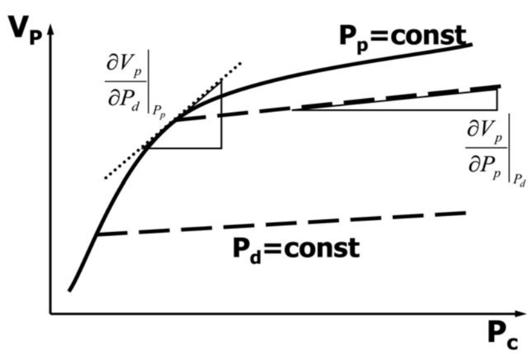

Figure 1 schematic behavior of compression-wave velocity with respect to confining pressure. (Hoffmann et al. 2005).

Pore Pressure Calculation

In this work, our objective is to test the sensitivity of effective pressure in reservoir section considering stress coefficient for Biot and Willies (1957), Todd and Simmons (1972) in comparison with Terzhagi (1923). The remaining parameters of overburden stress and pore pressure can be calculated using established methods in the past. In this work, we used a gradient of 1psi/ft for plotting overburden stress trend. For pore pressure calculation, we used Eaton's method for pore pressure prediction from compressive-wave velocities (equation 6). Eaton's method is one of the widely used algorithm in pore pressure prediction and is robust too. We used an exponent of 3, although values from 3 to 5 have been recommended for pore pressure prediction using V_p (Ebrom and Heppard, 2003 & 2007). We used 0.465 psi/ft as the hydrostatic pore pressure gradient value.

$$P_p = P_{ob} - (P_{ob} - P_{hy}) * \left(\frac{\Delta t_n}{\Delta t}\right)^3 \qquad (6)$$

Where, Δt= Sonic log values for P-wave, and $\Delta t_n = \Delta t_m + (\Delta t_{ml} - \Delta t_m)e^{-cZ}$, where Δt_m is the transit time in rock matrix, Δt_{ml} is the mudline transit time (Table 1), c is a constant and Z is the depth.

Results

In this paper, we examined the sensitivity of above detailed three methods for effective pressure prediction in shales and carbonate rocks. The coefficient (β) for Biot's method were calculated using the steps detailed above. The effective stress coefficient (α) were calculated using Figure 2 (Sharma, 2015). Figure 2 shows the compressive-wave velocity behavior at full brine saturation of the reservoir core under different combinations of confining and pore pressures. For this work, we choose a value of α = 0.827 close to the saturating values of 0.7 for higher confining pressure (deeper formations).

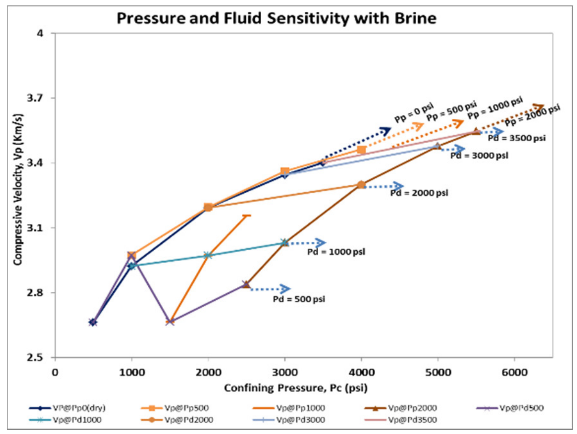

Figure 2 compressive-wave velocity (V_p, Km/s) plotted as a function of confining pressure (P_c, psi). Effective stress coefficient is obtained from the ratio of slopes at the intersection point of the constant pore pressure and constant differential pressure line (Sharma 2015).

We next calculated the pore pressure in the carbonate reservoirs and the shales. Figure 3a shows the pore pressure behavior. Pore pressure trend in shales was equal to or slightly lower than the hydrostatic gradient of 0.465 psi/ft, whereas most reservoir sections show high pore pressure. Effective pressure calculations using equation 1 and 2 are shown in Figure 3b. Effective pressure values from Terzhagi method in shales are trending along the normal effective pressure trends (red color). Biot's coefficient in shale predicted the effective pressure more than the other two methods and looks like under direct influence of the poor V_p log quality and probable lack of control on the elastic constants used for shale mineralogy. Effective pressure values in the reservoir are increasing with decrease in effective stress coefficient values. For these reservoirs, apparently the prediction using Biot's (β) and effective stress coefficient (α) is tied closer to each other than the prediction using Terzhagi (1923). We now wanted to understand the correlation between effective pressure and velocities. We understand that the actual causative mechanism is control of effective pressure on distribution of velocities in the reservoir, unfortunately there is no mechanism in place to reliably model these velocities based calculated effective pressure. Figure 4a and 4b show the effective pressure calculated using three different methods versus compressive and shear-eave velocity, respectively. In both the plots, we see that effective pressure calculated using 'α' overlaps more or less with the effective pressure using 'β' in the reservoir section (predominantly carbonates) particularly in the low velocity values. In the shale section plotted on the right side of the demarcation, experimentally controlled effective pressure overlaps more with Terzhagi values in shale. Particularly in the shale velocities, the value of 'β' appears as scattered for the reason explained above in this section.

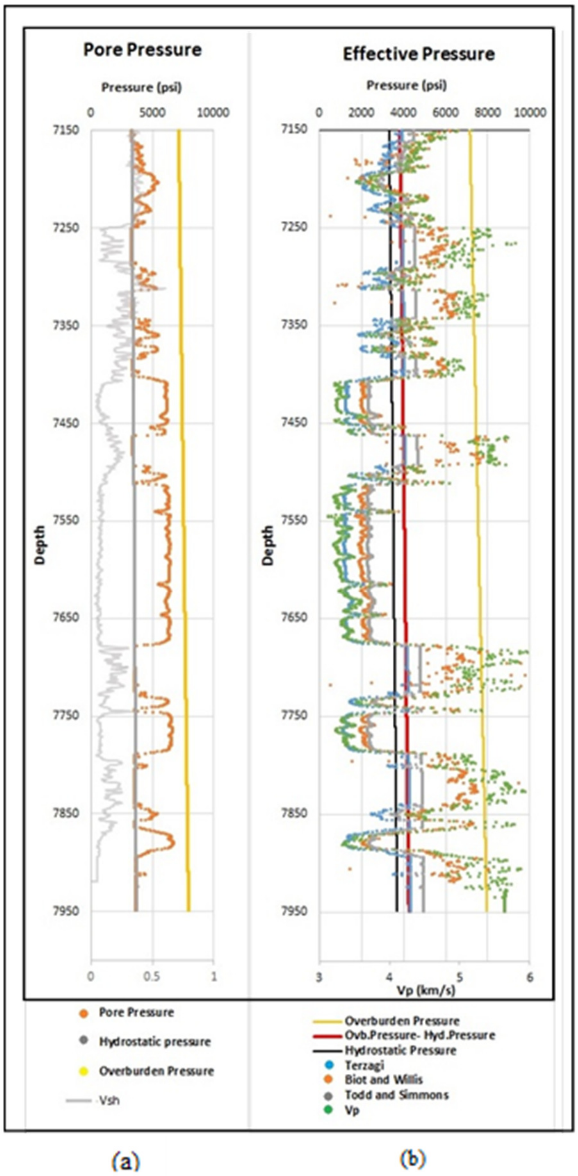

Figure 3 The left panel (a) shows the pore pressure distribution calculated using Eaton's equation and the shale volume distribution in light grey color. All the reservoir section show higher pore pressure. The right most panel (c) shows the three gradients: overburden gradient in yellow, pore pressure gradient in black, and effective pressure gradient in red. In addition this panel shows the measured compressive-wave velocity in green and the three calculated effective pressure: Terzhagi in blue, Biot and Willies in Orange, and the Todd and Simmons in grey color.

Conclusions

For non-clastic and non-granular rock such as carbonates in which compaction trends are generally missing, experimental data provide the accurate estimates of effective stress coefficients. Given the good quality of velocity data in the reservoir sections, the overlap of effective pressures based on α and β (two completely independent approach) suggest that assuming a Terzhagi trend for effective pressure prediction would be way off the mark. Whereas for the shale sections, we expected the three to converge as the values of the coefficients (α, β) used was close to 1. However, the scatter of 'β' in the shale section for obvious reasons of inferior P-wave log and a probable lack of control on elastic parameters of shale does not allow us to conclude convincingly on the reliability of effective pressure using β in the shale section. This is important observation as in the absence of experimental data, scientists usually opt for Biot's method for understanding effective pressure trends in the reservoir and shale.

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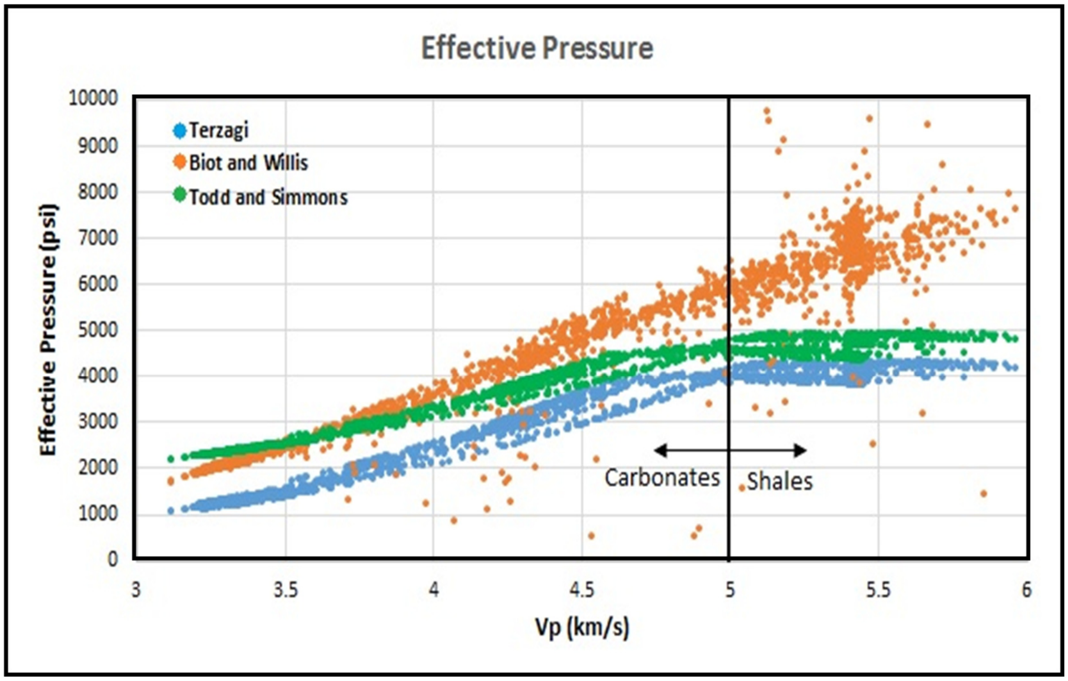
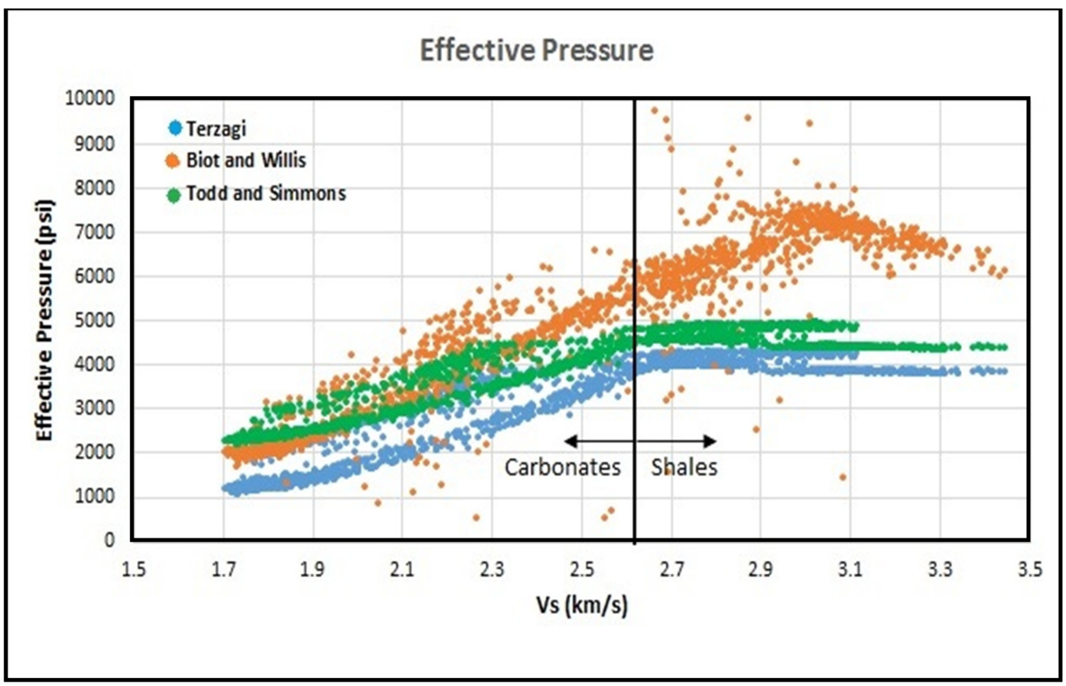


(a)



(b)

Figure 4 Effective pressure from three different methods versus (a) compressive-wave velocity and (b) shear-wave velocity. A clear demarcation is drawn in the two plots showing trends of effective pressure and velocity in shales and carbonates.