Use of Fractal Geometry for Determination of Pore Scale Rock Heterogeneity

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Summary

Fractal geometry provides new insight in mathematical description of heterogeneity observed in the pore structure of the reservoir rocks. The fractal dimension is used for description of fractal geometry; bigger the pore fractal dimension, the more heterogeneous the fractal object. This paper presents the fractal analysis of rock samples of Linch pay sand of Jotana field in Western Onshore Basin of India. In this study, fractal dimension is obtained by plotting the mercury saturation versus capillary pressure data generated from high pressure mercury injection studies. The heterogeneity obtained using this fractal approach has been verified with pore geometry analysis

Introduction

The effective reservoir description requires detailed knowledge of the heterogeneity, both at microscopic and macroscopic levels. The performance of a reservoir is controlled by certain properties of fluids and the geometry of the pore system. Accurate determination of pore-body/ throat attributes and fluid distribution are central elements in improved reservoir description.

In recent years, it is found that pore structure of sandstone or other porous media are fractal[1]. Angulo, Alvorado and Gonzalez[2] published a method by which mercury porosimetry data can be used to extract information about the fractal characteristics of the pore space of the rock sample.

In this paper fractal dimensions were calculated from high pressure mercury injection capillary pressure data generated on four core samples of Linch pay sand of Jotana field in Western Onshore Basin of India. Fractal dimensions of the pores were calculated based on models used by Angulo, Alvorado and Gonzalez[2] and Li Kewan[3].

Fractal dimension concept

The origin of the term fractal is due to the fact that they have a fractional dimension, not a whole number value. Where classical geometry deals with objects of integer dimension, fractal geometry describes non-integer dimension. Points have zero dimension, lines and curves have one dimension, squares and circles have two dimensions and cubes and spheres have three dimensions (Fig.1).

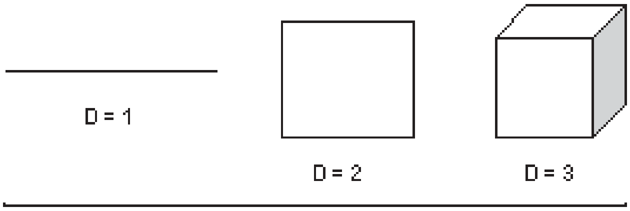

Fig. 1: Integer dimension of classical geometrical object

Consider an object residing in Euclidean dimension D and reduce its linear size by 1/r in each spatial direction, its measure (length, area, or volume) would increase to $N=r^D$ times the original (Fig. 2). Taking the log of both sides, and get log(N) = D log(r). Solving for D, D = log(N)/log(r), D need not be an integer, as it is in Euclidean geometry and it could be a fraction, as it is in fractal geometry. This

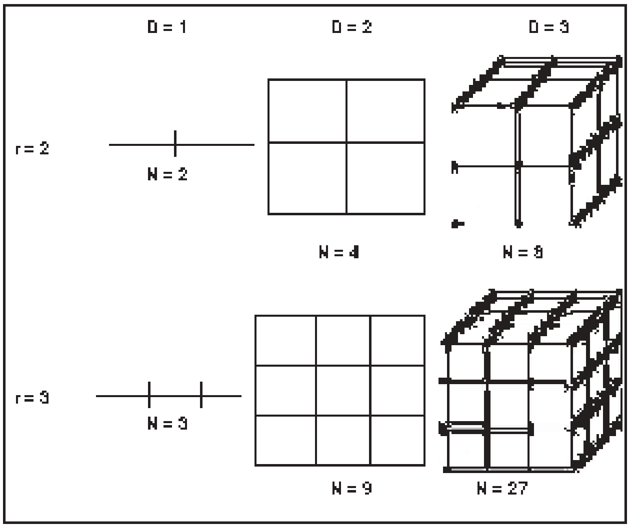

Fig. 2: Integer dimension of classical Geometrical Object reducing its linear size by 1/r in each spatial direction

generalized treatment of dimension is named after the German mathematician, Felix Hausdorff. It has proved useful for describing natural objects.

So, an important feature of a fractal object in nature is its self-affinity with a dimension that is fractal rather than integer. In general, self-affinity set of this type is expressed by the equation

$$N(r) \propto r^{-Df} \tag{1}$$

Where, r is the smallest dimension, N(r) is the number of the units with a dimension of (r) requires to fill entire object and Df is the so called fractal dimension.

The fractal dimensions are used in many applications to describe rugged boundaries and as well as heterogeneity of the object. Greater the dimension, the more heterogeneous the fractal object.

Fractal geometry of pore space

Capillary pressure data generated using mercury intrusion technique is considered to be most suitable to reveal the characteristics of pore structure in reservoir rocks. It is assumed that pore network of rock sample is made up of many pore tubes with different radius and the same length (the capillary tube model)[4]. The capillary tube model of the Washburn equation is generally used in order to calculate the pore size distribution of a rock sample from mercury injection capillary pressure data.

So from the capillary tube model, unit area (A) of rock sample can be represented by

$$A = N(r).r^{2} \tag{2}$$

Substituting Eq. 2 into Eq. 1:

$$A \,\alpha\, r^{(2-Df)} \tag{3}$$

Now, volume of the object is calculated by

$$V \,\alpha\, r^{(3-Df)} \tag{4}$$

So, if the pore space of a sample is fractal, then the intruded volume of mercury for mercury porosimetry with increasing pressure also will scale as a fractal[5]. So the Eq. 4 can be expressed as

$$V_{Hg} \,\alpha\, r^{(3-Df)} \tag{5}$$

Where, V_{Hg} is the cumulative volume of mercury intruded into the pores.

Considering capillary tube model of *Wasburn equation*, the capillary pressure can be expressed as follows:

$$P_c = (2\alpha \, Cos\theta)/r \tag{6}$$

Where, P_c is the capillary pressure, ó is the surface tension and è is the contact angle. Substituting Eq. 6 into Eq. 5:

$$V_{Hg} \,\alpha\, P_c^{-(3-Df)} \tag{7}$$

As α and α are constants during an experiment.

The mercury saturation is calculated as follows:

$$S_{Hg} = V_{Hg}/V_p \tag{8}$$

Where, S_{Hg} is mercury saturation and V_p is pore volume

Substituting Eq. 8 into Eq. 7:

$$S_{Hg} = aP_c^{-(3-Df)} \tag{9}$$

Where, a is the constant.

Taking the logarithm of this equation gives:

$$\log(S_{Hg}) = -(3-Df)\log(P_c) + C \tag{10}$$

So the fractal dimension can be obtained by drawing the mercury saturation versus capillary pressure on a log-log plot. This equation is similarly used by Kewen Li[3].

Experimental procedure

Four core samples belonging to Linch pay sand from one well of Jotana field were used for this study. Porosity was determined using *Helium Porosimeter* working on *Boyle's Law* of gas expansion method. Permeability to air was determined by *Permeameter* based on steady state flow of gases. The measured porosities are in the range of 12.36% to 28.21% and permeabilities in the range of 5.78 millidarcy to 901.47millidarcy (Table-1).

High Pressure Mercury Porosimeter Autopore II 9215 capable of injecting mercury upto 55000 psi has been

Table 1: Fractal dimension of rock samples

Sample No.	Porosity (%)	Permeability to air (millidarcy)	Fractal dimension, Df
1103	24.14	86.05	3.119
1204	24.94	5.78	3.247
2403	28.21	901.47	3.058
2503	12.36	8.36	3.134

used to generate pore size distribution data. This apparatus works on the principle of mercury injection for determining the pore aperture size distribution in porous rocks based on Washburn equation (**Eq. 6**). The instrument used for the study performs automatic injection of mercury at programmed steps from less than 1 psi to 55000 psi. The experiment runs for pressure upto 25 psi at low pressure run and then high pressure run i.e. 25 psi to 55000 psi.

Results

Fractal dimension were calculated from high-pressure mercury injection capillary pressure data using Eq.10. The calculated fractal dimensions are given in Table-1 and plots were given Fig.3 to Fig.6.

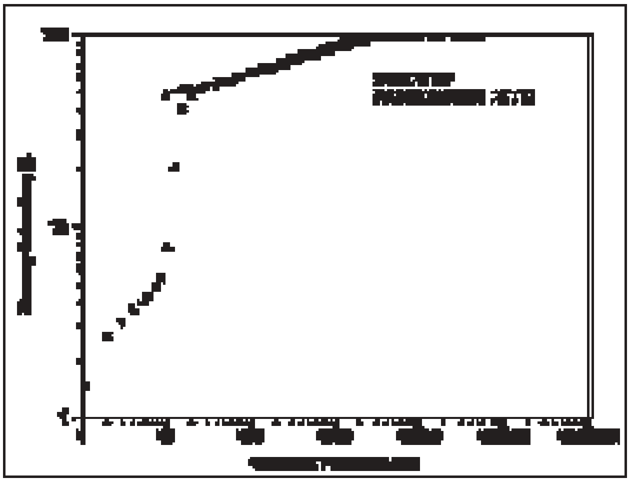

Fig. 3: Relationship between mercury saturation and capillary pressure of sample 1103

The calculation of fractal dimensions was based on the linear part of the plots. The derived values of the fractal dimensions of the four samples have following order: Df(Sam-2403)<Df(Sam-1103)<Df(Sam-2503)<Df(Sam-1204). The sample with greater fractal dimension has greater heterogeneity. So, sample-2403 is the most homogenous and sample-1204 is the most heterogeneous.

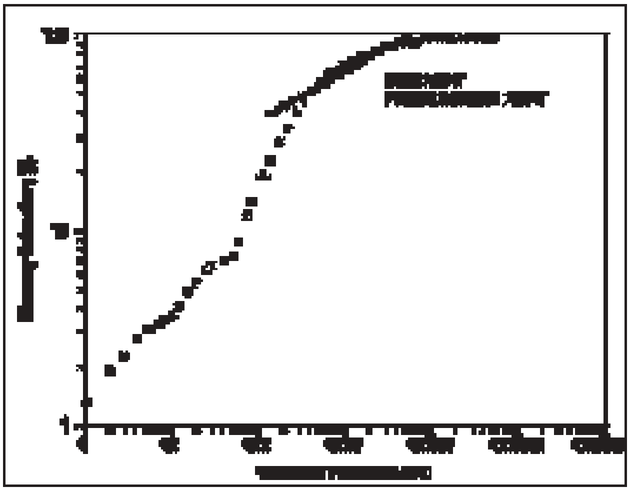

Fig. 4: Relationship between mercury saturation and capillary pressure of sample 1204

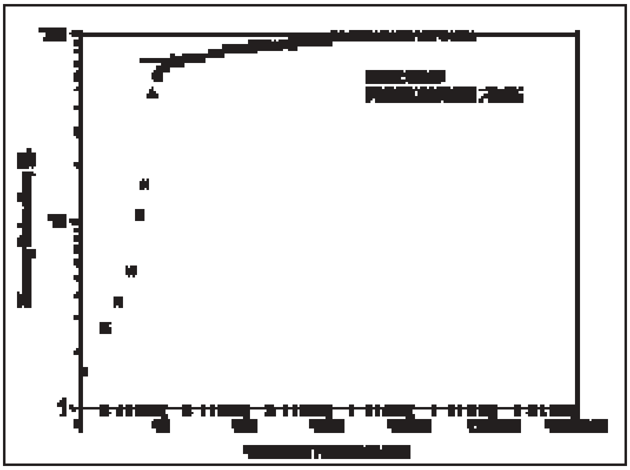

Fig. 5: Relationship between mercury saturation and capillary pressure of sample 2403

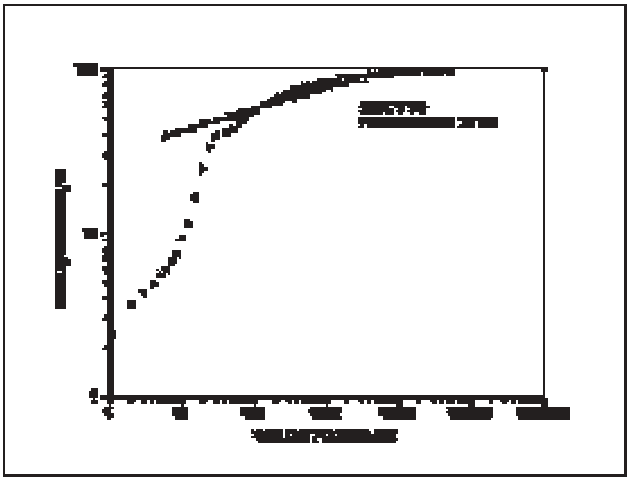

Fig. 6: Relationship between mercury saturation and capillary pressure of sample 2503

Additional information on the heterogeneity of the samples is provided by pore size distribution study. Based on high-pressure mercury injection capillary pressure data, the pores of samples were divided into 3 units according to their pore radius (r) as macro, meso and micro. **Table-2** shows the relative abundance of macro, meso and micro pore for their respective samples.

Table 2: Pore size distribution for rock samples

Sam. No.	**Macro pores r >1.5μ (%PV)**	**Meso pores 0.5μ <r<1.5μ (%PV)**	**Macro + Meso (%PV)**	**Micro poresr < 0.5μ (%PV)**
1103	56.37	11.76	68.13	31.87
1204	7.04	21.35	28.39	71.61
2403	79.89	7.33	87.22	12.78
2503	42.44	14.56	63.57	36.43

The most homogenous sample (2403) had the higher ratio of macro plus meso to micro pores. The most heterogeneous sample (1204) showed wider distribution of pore throat sizes than homogenous sample, which is dominated by micropores. Fig.7 to Fig.10 show pore size distribution studies of four samples. The most homogenous sample (2403) shows the pore sizes in a normal distribution with a relatively narrow peak (Fig.9). However, the pore size distribution of the most heterogeneous sample (1204) does not show normal distribution and has many peaks (Fig. 8).

Rock heterogeneity can also be characterized by using Pore Level Heterogeneity Index (Hi). Jones[6] has shown that $\beta K_\infty \phi$ is the proper characteristic length for a porous medium in the Reynold's number expression. From fundamental principles, Jones showed that for a homogenous core plug system, $\beta K_\infty \phi$ will tend to approach the value of the mean hydraulic radius ($\sqrt{K/\phi}$). Therefore, the deviation of $\beta K_\infty \phi$ from ($\sqrt{K/\phi}$) can be used as a heterogeneity indicator on a core plug level[7]. The heterogeneity index, Hi, is defined as follows:

$$\alpha = 3.238 \times 10^{-9} \beta K_\infty \qquad \text{———— (11)}$$

$$\text{Mean hydraulic radius } (R_{MH}) = 0.0314\,(\sqrt{K/\phi}). \qquad \text{——— (12)}$$

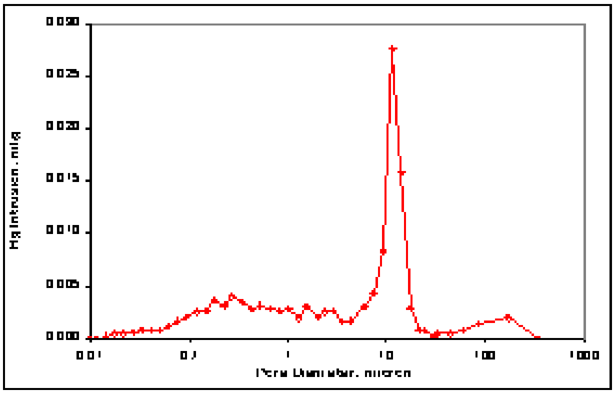

Fig. 7: Incremental Intrusion VS Pore Diameter of Sample No. 1103

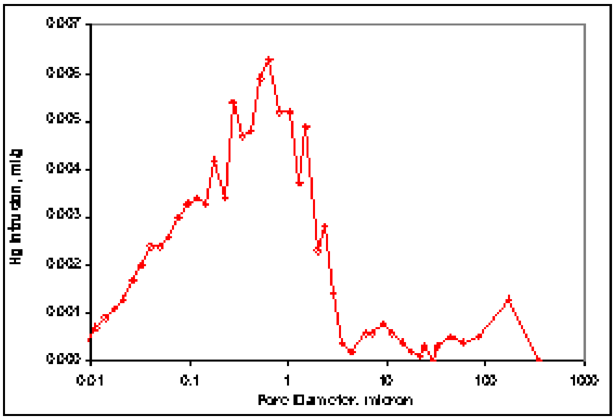

Fig. 8: Incremental Intrusion VS Pore Diameter of Sample No. 1204

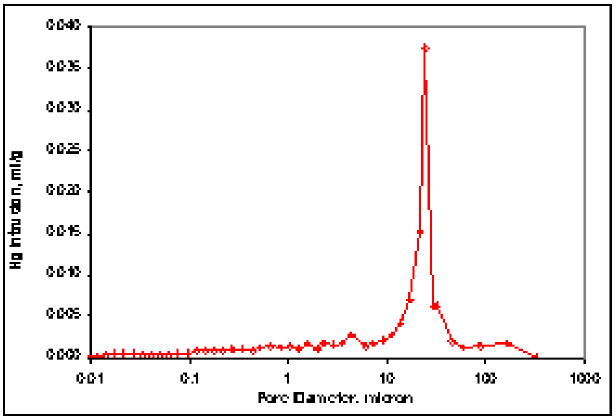

Fig. 9: Incremental Intrusion VS Pore Diameter of Sample No. 2403

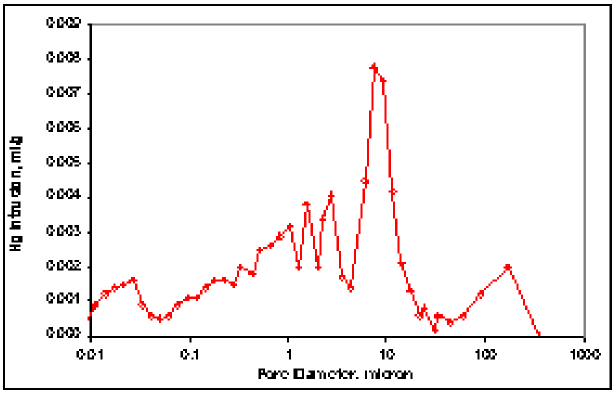

Fig. 10: Incremental Intrusion VS Pore Diameter of Sample No. 2503

$$Hi = \text{Log}_{10}\,(\alpha\phi/R_{MH}) \quad (13)$$

Where,

β = Forchheimer inertial resistance.
= $4.2X10^{10}K^{-1.35}$ Katz, et al.[8]
K_∞ = Klinkenberg permeability

For a completely homogenous system, the ratio ($\alpha\phi/R_{MH}$) would be 1 and index, Hi, will be zero.

Table-3 shows Hi values of different samples. From the table it is indicated that sample-2403 has lower value of Hi and the most homogenous while sample-1204 has a higher value of Hi and the most heterogeneous.

Table 3: Heterogeneity index of rock samples

Sample No.	Heterogeneity index(Hi)
1103	**1.11**
1204	**2.14**
2403	**0.34**
2503	**1.54**

Conclusions

1. Fractal dimension calculated from laboratory derived pore size distribution data could be quantatively used to characterize the heterogeneity of rock samples. Greater the fractal dimension, the more heterogeneous the rock.
2. The study concerning pore level heterogeneity index is also supported by fractal analysis. Lower value of heterogeneity index and smaller fractal dimension are indicative of greater homogeneity in rock samples.

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