



# A Simple Approximation to the P-wave Reflection Coefficient and its Implication in the Amplitude Variation with Offset (AVO) Data for Gas Hydrate Identification

Uma Shankar\*, N. K. Thakur and Ramesh Khanna

National Geophysical Research Institute, Hyderabad

E-mail: [umashankar\\_ngri@yahoo.com](mailto:umashankar_ngri@yahoo.com)

## Summary

The amplitude of reflected seismic waves are directly proportional to seismic reflection coefficients at an interface between different geological strata below the earth's surface. The theory of AVO is based on the Zoeppritz equations, which give the plane wave reflection and transmission coefficients as a function of angle of incidence and of six independent elastic parameters (i.e. P-wave and S-wave velocities in upper and lower medium and densities of the medium) three on each side of the reflection interface. These changes may be used to infer fluid or lithologic changes from one geologic formation to another. The variation of reflection amplitude-versus-offset (AVO) or amplitude versus incidence angle can be an important parameters in arriving at petrophysical / lithological models. The patterns of AVO behavior may put important constraint to work out models for generation of Bottom Simulating Reflector (BSR), a prime indicator for the gas hydrate identification on seismic sections. The objective of the present work is to study the variation of reflection coefficient with angle of incidence for shale / gas sands models with gas hydrates. The study shows variation in the reflection coefficient obtained by approximate and exact solution. Different models have been considered for study of the variation of P-wave reflection coefficient with angle of incidence and some of the results are presented.

## Introduction

Amplitude Variation with Offset (AVO) analysis of seismic reflection data has been of growing interest to exploration seismologist over the past few years. In fact, P-wave AVO from pre-stack gathers is now used routinely to infer the presence of hydrocarbon in some geological settings. The theory of AVO is based on the Zoeppritz equations, which give the plane-wave reflection and transmission (R/T) coefficients as function of angle of incidence and of six independent elastic parameters, three on each side of the reflection interface. The exact mathematical expression for R/T coefficients is complicated (Koefoed, 1962, Aki and Richard, 1980).

For a stacked seismic section, the P-wave arrives normal to the interface between two different rock types. The incident energy is both reflected and transmitted at the interface as a P-wave whose strength depends upon the elastic properties of the different rock types. However, for a non-stacked common depth point (CDP) gathers, the seismic P-wave strikes obliquely at the interface. The seismic energy generates both P-waves and S-waves, which are both reflected from and transmitted through the interface. This phenomenon of mode conversion results in amplitude variations across offset in a seismic CDP gather.

The reflected P-wave amplitude of oblique incidence is a function of P-wave velocity, S-wave velocity, and density of each rock for a given angle of incidence. P-wave velocities and densities can be measured from well logs. However, S-wave velocities are empirically determined or estimated from studies on Poisson's ratio (Hamilton, 1976, Gregory, 1976, Domenico, 1976, 1977). Then P-wave reflection can be calculated for various incidence angles by solving Zeoppritz's amplitude equation with explicit expressions (Zeoppritz's, 1919, Young and Braile, 1976)

Gas hydrates are ice like crystalline structures formed at very low temperature and high pressure in marine environment. Apparently gas hydrates cements sediments, and therefore, it can have a significant effect on sediment strength, its formation and break down may influence the occurrence and location of submarine landslides. Such landslides may release methane into the atmosphere, which may affect global climatic change.

Although gas hydrate has been recognized in drilled cores, its presence over large areas can be detected much more efficiently by acoustical methods, using seismic reflection profiles. Hydrate has a very strong effect on acoustical reflection because it has a high acoustic velocity (approximately 3.3 km/s about twice that of sea-floor

sediments), thus grains cemented with hydrate produce a high velocity deposit due to the mixing of hydrate with the sediments. The high impedance contrast produced by the hydrate-cemented sediments above and low velocity zone due the sediment saturated with water (water velocity 1.5 km/s) or free gas produces strong reflection at the interface. Because the base of the gas hydrate stable zone occur at an approximately uniform sub-bottom depth throughout a small area, this well-defined seismic reflection from the base of the zone roughly parallels the sea floor and is called the Bottom Simulating Reflector (BSR).

A second significant seismic characteristic of hydrate cementation is called “blanking”. Blanking is the reduction of the amplitude (strength) of seismic reflection that apparently caused by cementation by hydrate of the strata that form reflectors. The blanking effect occurs throughout the entire hydrate-cemented zone and can be quantified to estimate the amount of gas hydrate that is present. Over large area the thermal field may be distorted and even the pressure field and the chemistry of the pore water may vary unpredictably, all are factors that affect the phase stability of hydrate. Therefore the variation in depth to the base of the gas hydrate stable zone can indicate a great deal about sub-sea-floor conditions.

## Methodology

In the present work an attempt has been made to study variation of P-wave reflection coefficients with angle of incident. The variation of reflection coefficients & transmission coefficient with angle of incident has been studied using Zoeppritz equations (1919) and there approximations have been proposed. e.g.; Aki and Richards (1980), Shuey (1985) and Mallick (1993). These approximations have been proposed for interpreting AVO data over gas sands and gas hydrate models.

Hyndman and Spence (1992) show the effect of AVO studies over gas hydrates. Amplitude of the gas hydrate varies as offset varies. To see the variation of reflection coefficients with angle of incident they used the Shuey’s approximation. Exact form of Zoeppritz equation may provide information of the variation of P-wave reflection and transmission coefficients with angle of incident.

Approximation of P-wave reflection coefficients given by AKI and Richards (1980)

$$R(i) = \frac{1}{2\cos^2(i)} I_P - 4\gamma^2 \sin^2(i) I_S + (2\gamma^2 \sin^2(i) - 0.5 \tan^2(i)) D$$

Where

$$I_P = \left( \frac{\Delta V_P}{V_P} + \frac{\Delta \rho}{\rho} \right), I_S = \left( \frac{\Delta V_S}{V_S} + \frac{\Delta \rho}{\rho} \right), D = \frac{\Delta \rho}{\rho}$$

With,

$$\Delta V_P = V_{P2} - V_{P1}, \Delta V_S = V_{S2} - V_{S1}, \Delta \rho = \rho_2 - \rho_1$$

$$V_P = (V_{P2} + V_{P1})/2, V_S = (V_{S2} + V_{S1})/2, \rho = (\rho_2 + \rho_1)/2$$

Approximation given by Shuey (1985)

$$R_{(i)}/R_{(\theta)} = 1 + A \sin^2 i + B(\tan^2 i - \sin^2 i)$$

Angle of incidence (i) is the average of incidence and transmission angles, which is given as:

$$i = (i_1 + i_2) / 2$$

$$\Delta \sigma = (\sigma_2 - \sigma_1)$$

$$\sigma = (\sigma_2 - \sigma_1) / 2$$

Now,

$$V_S^2 = V_P^2 (1 - 2\sigma) / (1 + \sigma)$$

Where,

$$R_0 = 1/2 (\Delta V_P / V_P + \Delta \rho / \rho), A = A_0 + \Delta \sigma / [(1 - \sigma)^2 R_0]$$

$$A_0 = B - 2(1 + B)(1 - 2\sigma) / (1 - \sigma),$$

$$B = (\Delta V_P / V_P) / (\Delta V_P / V_P + \Delta \rho / \rho)$$

## Examples

We present two examples comparing approximate and exact solutions of the reflection coefficient for AVO behavior. These examples cover different possible ways the P-wave reflection coefficient can vary with angle of incidence. In all the examples below,  $V_{p1}$ ,  $V_{s1}$  and  $\rho_1$  denotes P-wave velocity, S-wave velocity and density respectively, for the medium above the reflecting interface, where as  $V_{p2}$ ,  $V_{s2}$  and  $\rho_2$  denotes the same quantities for the medium below the reflecting interface. In the following examples, we therefore compare the exact reflection coefficient with the approximate reflection coefficient given by shuey (1985) and Aki and Richards (1980).

Examples 1 is for model with  $V_{p1}=2271$  m/s,  $V_{p2}=1920$  m/s,  $\rho_1=2.15$  g/cm<sup>3</sup>,  $\rho_2=1.95$  g/cm<sup>3</sup>. This choice of parameters correspond to  $V_{s1}=1279.8$  m/s and  $V_{s2}=927.2$  m/

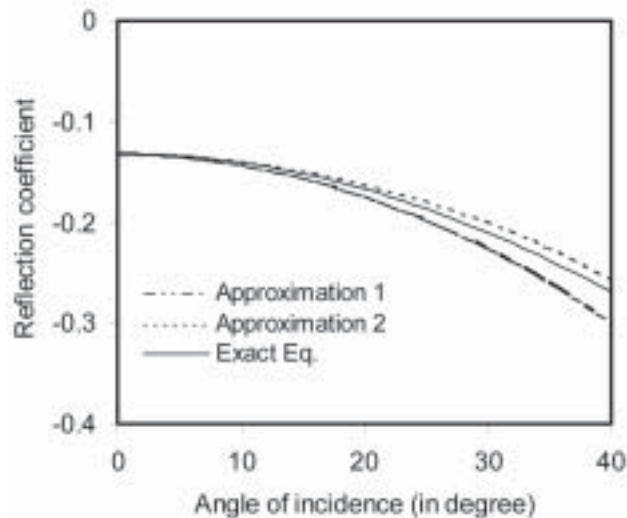


s. The example represents an interface between shale and gas sand. Computed exact and approximate reflection coefficients for this model are shown in Figure 1. Note that the exact and the approximate reflection coefficients for this model are so close that they overlay each other in Figure 1. This example in fact the same as the one used by Shuey (1985).

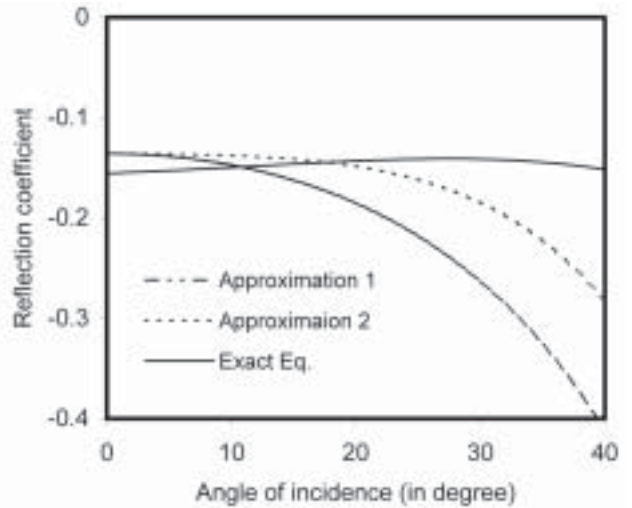
Examples 2 is for model with  $V_{p1}=3800$  m/s,  $V_{p2}=1500$  m/s,  $\rho_1=2.19$  g/cm<sup>3</sup>,  $\rho_2=1.72$  g/cm<sup>3</sup>. The computed shear velocities are  $V_{s1}=1358.6$  m/s and  $V_{s2}=248.3$  m/s. The P-wave reflection coefficients are shown in Figure 2. This example can be representative of an interface between gas hydrate and seafloor sediment.

The objective of present work is to study the variation of reflection coefficient with angle of incidence for gas hydrates and gas sand. Another objective is to study the variation in the reflection coefficient obtained by approximate solution from exact equation. The reflection coefficient depends upon the P and S wave velocities, density of the medium and angle of incidence. The variation of amplitude versus offset (AVO) or reflection coefficient versus incidence angle can be important indicator of free gas at interface. Two models have been study of the variation of P-wave reflection coefficient versus incidence angle. First model contain shale-gas interfaces and second one is gas hydrate-gas sand interface. The range of angle of incidence used in the study is from 0° to 40°.

In the following graphs, curves showing variation of p-wave reflection coefficient with angle of incidence



**Fig. 1:** P-wave reflection coefficient versus angle of incidence plot using three equations for the shale and underlying gas sand.



**Fig.2:** P-wave reflection coefficient versus angle of incidence plot using three equations for the gas hydrate and underlying gas sand.

corresponding to Zoeppritz equation have been termed exact solution whereas those based on formula of Aki and Richards (1980) have been called first approximate solution or first approximation. Those corresponding to formula of Shuey (1985) have been called second approximate solution or second approximation.

The results for model 1 are shown in Figure 1. The reflection coefficients assume larger (negative value) with increasing angle of incidence. For this model, the contrast in P and S wave velocity is not very high. The value of reflection coefficient for exact solution is in the range – 0.13 to -0.34. In this case, the exact solution and those corresponding to second approximation are very close. The third curve based on the first approximation coincide with the other two curves for small angle of incidence (up to 10°) and then deviates giving smaller (negative) values. When angle of incidence approaches 50°, this curve comes closer to the other curves. In this case, the maximum deviation is very small from the exact results. Thus it can be concluded that at least for this or similar model, any one of the formula can be used. The difference between exact solution and the two approximations is not significant, so any of these can be used.

The gas hydrates are high velocity sediments. The sediment below the gas hydrate layer has low velocity, which is due to the presence of free gas. Since, gas hydrates are thick layer of solid crystals, so that the wave velocity in the medium is increased. The velocity contrast in this model is large. Due to the large Poisson's ratio in the lower medium,

the S-wave velocity is very small. The reflection coefficients for exact solution increase initially then decrease. The difference in the values of reflection coefficient changes with the angle of incidence. Initially both the approximations have nearly same values up to 4 but at larger angle of incidence i.e. 15-30, the difference is increased. The maximum deviation is about 14%. From curve's it can be concluded that approximation 2 is much closer to the Zoeppritz solution than the approximation 1 (Fig. 2).

## Conclusions

The difference in the results between exact solution and two approximate solutions is very small in case of shale / gas sand models considered in the present study, so that any one of them can be used for the computation of P-wave reflection coefficients. The results for the two approximations are close to each other for gas hydrates and curves based on exact solutions are very different from the two approximations. In case of gas hydrate model, the reflection coefficients initially decrease slightly with angle of incidence and beyond 40° rapid increase of reflection coefficient with angle of incidence, so that we can say that at the interface of the model presence of gas.

## Acknowledgments

The authors wish to thank Dr. V. P. Dimri, Director, National Geophysical Research Institute, Hyderabad, for his kind permission to publish this paper and Gas Authority of India Ltd. (GAIL) for making data available to us.

*The views expressed in this paper are of the author(s) only and not necessarily of ONGC.*

## References

- Aki, K., and Richards, P. G., 1980, Quantitative seismology: Theory and methods: W. H. Freeman and Co.
- Domenico, S. N., 1976, Effect of brine-gas mixture on velocity in an unconsolidated sand reservoir, *Geophysics*, 41(5), 882–894.
- Domenico, S. N., 1976, Elastic properties of unconsolidated porous sand reservoir, *Geophysics*, 42, 1339–1368.
- Gregory, A. R., 1976, Fluid saturation effects on dynamic elastic properties of sedimentary rocks. *Geophysics*, 41, 895–921.
- Hamilton, E. L., 1976, Shear wave velocity versus depth in marine sediments. *Geophysics*, 41, 985–996.
- Hyndman R.D. and Spence. G.D., 1992, A seismic study of methane hydrates, Marine bottom simulating reflectors, (1997), *Jour. Geophysical Res.*, B5, 6683–6698.
- Koefoed, O.: 1962, Reflection and transmission coefficients for plane longitudinal incident waves: *Geophysics. Prosp.*, 10,304351.
- Mallick Subhashis, 1993, A simple approximation of the P-wave reflection coefficient and its implication in the inversion of amplitude variation with offset data, *Geophysics*, 58, 552–554
- Shuey, R. T., 1985, A simplification of the Zoeppritz equations: *Geophysics*, 50, 609–614.
- Young, G. B., and Braile, L. w., 1976, A computer program for the application of Zeoppritz's amplitude equations and Knott's energy equation. *Bull. Seis. Soc. Am.*, 66,1881–1885.
- Zoeppritz, K., 1919, *Erdbebenwellen VIII B*, On the reflection and penetration of seismic waves through unstable layers: *Goettinger Nachr*, 66–84.