Inversion of Azimuth Dependent Seismic Velocities and Amplitudes to Characterize Fracture Geometry and Fluid Infill

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Summary

Presence of natural fractures in hydrocarbon reservoirs results in directional variation of permeability. As the natural fractures have apertures much smaller than the wavelength of seismic waves used in exploration, the observable seismic attributes are velocity anisotropy, shear wave splitting and variation of reflection amplitudes with offset and azimuth (AVOA). Fracture attributes, viz. Fracture orientation, fracture density and fracture infill can be estimated from the analysis of AVOA data. Knowledge of fracture attributes serves as key parameters for drilling operation in a fractured reservoir for enhanced recovery of oil and gas.

In the present study, we investigated the nature of amplitude variation with offset and azimuth (AVOA) for pure and mode converted waves from a fractured media with different fluid saturation. The velocity and reflection coefficients have direct relations with fracture orientation, fracture density. By sorting over azimuth and offset in a gather, the azimuthal variation of velocity as well as amplitudes can be determined. Quantitative relations between approximate reflection coefficients (PP and PS) and excess fracture compliances have been established and used to extract information about fracture attributes from surface seismic data. For this purpose a general slip model has been used to represent the elastic properties of fractures. It has been shown that both the PP and PS reflectivity are sensitive to normal and shear fracture weaknesses- dimensionless parameters simply related with the excess fracture compliances. The ratio of the normal and shear weakness is a measure of the fluid content in the reservoir. The experimental results reveal that for small fracture weakness (15% or less), inversions of NMO velocities as well as pure mode P-wave AVOA data yield reliable estimates of fracture weakness and fluid saturation.

Most of the investigations on seismic wave propagation in fractured media consider vertical or steeply dipping fractures. Angerer (2002) reported the existence of obliquely dipping fractures in hydrocarbon reservoirs. Grechka and Tsvankin (2004) developed a method for quantitative characterization of obliquely dipping fractures. In an earlier work (Shaw and Sen, 2004), they used Born approximation to write an approximation of linearized PP-reflection coefficients over a transversely isotropic medium with tilted axis of symmetry (TTI) in terms of weak anisotropic parameters.

We have also investigated effects of fluid infill on the AVOA over obliquely dipping fractures since variation of seismic amplitudes with azimuth provides information on the strike of the fractures, the dip of the fractures can not be estimated easily. The presence of obliquely dipping fractures can be inferred from the following observations on single/multi component seismic data as (i) asymmetry of travel times in mode converted PS-waves (Angerer et al, 2002) reflected from a horizontal reflector.(ii) a significant horizontal component of P-wave as well as vertical component of slow S-wave, for near vertical incidence (Grechka and Tsvankin, 2004).

We also investigated the problem of attenuation of seismic waves in a visco-elastic medium. The results show that for an accurate amplitude analysis, the effect of attenuation need to be removed.

Introduction

Presence of natural fractures in an otherwise isotropic medium causes anisotropy in the elastic properties, on the scale of seismic wavelength. Besides splitting of shear waves, the speeds of propagation and the amplitudes of both compressional and the shear waves vary with the azimuth of the observation line with respect to the symmetry axis of the medium. For example, a fractured reservoir is sensitive to compressional wave across the fracture, because the incompressibility (k) becomes low reducing the P-wave velocity considerably. Gas filled fractures amplifies this effects allowing easy detection of gas columns. Further, shear wave splitting along the fractures is larger for gas filled fractures compared to fluid filled fractures (Grechka et al., 2003).

As fractures or cracks are planar discontinuities in a continuous rock medium, these not only provide space for storage of hydrocarbon but also allow relatively easier vents for flow of hydrocarbon, even if the medium has an intrinsic low permeability. Thus, the presence of fractures in a reservoir makes the permeability direction dependent. Knowledge of the direction of maximum permeability helps reservoir engineers plan for enhanced hydrocarbon recovery. Fracture characterization, i.e., estimating the attributes of fractures, viz., fracture orientation, dip and density are gaining importance for reservoir development. Conventional methods of characterizing fractures include analyses of cores, drill cuttings, downhole images, well logs etc. Over the last decade, there has been significant progress in understanding the effects of fractures on seismic data observed on surface as well as in boreholes, viz. vertical seismic profiles, VSP (Shaw, Sen and Chatterjee, 2005)

The presence of vertical fractures in an isotropic medium renders its elastic properties to become transversely isotropic with symmetry axis normal to the fracture surface, i.e., horizontal. Such media are termed as horizontal transversely isotropic (HTI). Both NMO velocity as well as amplitude variation with offset (AVO) over a HTI medium depends on the azimuth of the seismic line with respect to the symmetry axis (Ruger, 1998). The orientation and weak anisotropy parameters (Thomsen, 1986) can be estimated from the observed PP-AVOA data.

Though there are several models for representing elastic properties of fractures, Bakulin et al. (2000) showed that under the assumption of thin crack density and for wavelengths of seismic waves used in hydrocarbon exploration, all these models are appropriately represented by the general slip theory of Schoenberg and Sayer (1995). I used Born formulization (Shaw and Sen, 2004) to derive the sensitivity of PP and PS reflection coefficients on the fracture weaknesses. A set of synthetic PP-AVOA data has been inverted over a vertically fractured medium containing dry, partially filled and fully saturated fractures to estimate the normal and fracture weaknesses.

The apertures of cracks and fractures, ranging less than 0.5 micron for micro-cracks through a few mm for macro cracks and even up to few cm in some cases, is far smaller than the wavelength of seismic waves (~ 10-100m) in a common seismic experiment. However, the elastic properties of the fractures whether dry, partially or fully saturated with water, oil or gas are different from the otherwise isotropic rock matrix in which these are embedded. As a result, certain attributes of the seismic reflection, viz., the normal move out velocity (NMO), AVO etc. become dependant on the azimuth of the seismic line with respect to the symmetry axis of the fracture system. The other important effect of this '*seismic anisotropy*' is the 'splitting of the shear wave', i.e., shear wave with polarization parallel and perpendicular to the fracture strike travel with different velocities. In this work, we investigated how the analyses of azimuthal variation of NMO velocity and amplitude variation with offset and azimuth (AVOA) for P-waves can be used to estimate the fracture orientation, dip and fluid infill.

Methodology

When a seismic wave is incident on an interface separating two media with different elastic constants and densities, besides reflection and transmission, there is conversion of waves with different modes of propagation (Figure 1). The partitioning energy is governed by the continuity of displacement and stress at the boundary leading to famous Zoeppritz equations (Aki and Richards, 2002). The most important aspect of the effects of this partitioning of energy on exploration seismology is that both the reflection and transmission coefficients of a pure as well as mode converted waves vary with the angle of incidence (and hence on the offset between the source and the receiver). Under the condition of small changes in material properties across an interface, a condition surprisingly satisfied in most hydrocarbon bearing sedimentary environs, the complicated Zoeppritz equation reduces to a very simple form which is linear in the contrasts in the rock properties across the interface well below the critical angle of reflection.

Several forms of the linearized reflection coefficient (Bortfield, 1961; Richards and Frazier, 1976; Ostrander, 1984; Shuey, 1985) have been used in the hydrocarbon industry to predict lithology and fluid content in the reservoirs under the head "Amplitude Variation with Offset", AVO. Recently, Shaw and Sen (2004) have shown that linearization of the reflection coefficient is equivalent to considering the singly scattered waves from an array of scatterers embedded in an isotropic background medium. This method provides a very simple way to investigate into the effect of fractures on AVOA in seismic reflection data.

To achieve the goal, we represent the elastic properties of a set of vertical fractures embedded in an otherwise isotropic medium by two parameters, viz., the normal and tangential excess compliances (ratio of strain and stress), ΔN and ΔT respectively. The medium is now characterized by a symmetry plane (X-Z), an isotropy plane

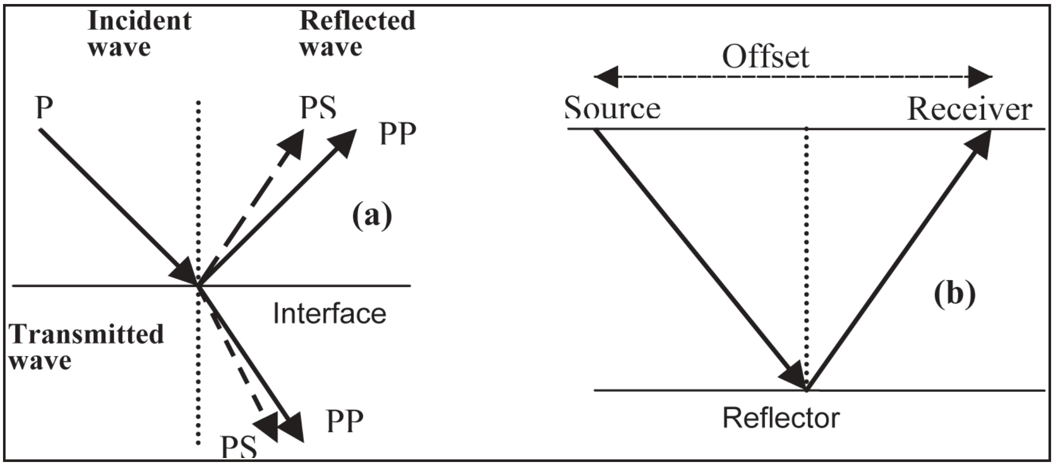


Fig. 1 : (a) A plane P wave incident at an interface separating two media with different elastic constants and/ or densities generates, in general, four waves, viz. reflected P, reflected S, transmitted P and transmitted S. The ratio of the reflected wave amplitude to the incident wave amplitude is called the reflection coefficient. (b) Variation of source receiver offset amounts to variation in the angle of incidence.

(X-Y) and a (rotational) symmetry axis along X-direction (Figure 2). As the overall elastic properties of the medium are transverse isotropic with the horizontal symmetry axis, such a medium is called Horizontal Transverse Isotropic (HTI). A seismic wave incident on such an interface finds the material properties to vary with the angle between the plane of incidence and the symmetry plane, resulting in azimuthal variation of reflection amplitudes. Following, Shaw and Sen (2005) we write the expression for P-wave AVOA as

ϕ

$$R(i,\phi) = A + B \sin^2 i + C \sin^2 i \tan^2 i \qquad \dots (1)$$

where, i is the angle of incidence and ϕ is the azimuth of the seismic line with respect to the axis of symmetry of the HTI medium, i.e. a direction perpendicular to the set of vertical fractures. The constants A, B, and C are expressed in terms of the P-wave acoustic impedance Z (product of density and velocity), shear modulus G, P-wave velocity α and the excess fracture compliances and as

$$A = \frac{1}{2}\frac{\Delta Z}{\bar{Z}} + \frac{1}{4}(1-2g)^2\,\Delta N\,, \qquad \dots (2a)$$

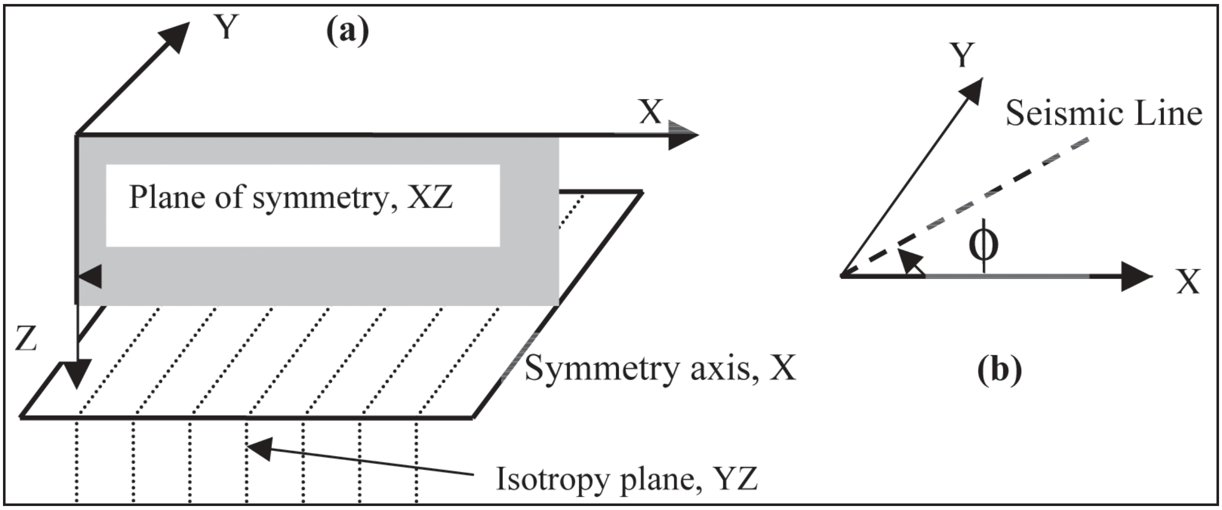


Fig. 2: (a) Presence of a set of vertical fractures in an isotropic medium, converts it into a transversely isotropic medium with horizontal axis of symmetry (HTI), (b) azimuth is the angle, between the seismic line and the symmetry axis measured in a horizontal plane.

$$B=\frac{1}{2}\left\{\frac{\Delta\alpha}{\bar{\alpha}}-4g\frac{\Delta G}{\bar{G}}\right\}+\frac{1}{4}\{(1-2g)+2g(1-2g)\cos 2\phi\}\Delta N-\frac{1}{2}g(1+\cos 2\phi)\Delta T \quad \ldots(2b)$$

and

$$C=\frac{1}{2}\frac{\Delta\alpha}{\bar{\alpha}}+\frac{1}{4}\left\{(1-2g+\frac{3}{2}g^2)+2g(1-2g)\cos 2\phi+\frac{1}{2}g^2\cos 4\phi\right\}.\Delta N+\frac{1}{8}g(1-\cos 2\phi)\Delta T \quad \ldots(2c)$$

with $g=\left(\frac{\bar{\beta}}{\bar{\alpha}}\right)^2$, β being the S-wave velocity in the medium. Here, the prefix and the over bar represent the contrast (difference) and the background (average) value of the physical parameter across the interface.

Equation (1) defines (co)sinusoidal variation of reflection amplitude with azimuth over a HTI medium and was first observed by Mallick and Frazer (1991) while simulating wave propagation in anisotropic medium. Ruger (1998) derived an analytic expression similar to equation (1). However, the equivalent to equation (2) in Ruger's formulation used weak anisotropy parameters ε, δ and Υ (Thomsen, 1988) to describe the elastic medium rather than using the excess fracture compliances and . Equations (1) and (2) can be used for characterizing the fractures, viz., estimating orientation and fluid saturation. It is worth mentioning that the ratio $\frac{\Delta N}{\Delta T}$ provides a quantitative measure of the fluid saturation in the fractures. This ratio of the normal to shear excess compliances of a (planar) fracture assumes value close to zero for gas filled fractures and tends to unity as the fractures are saturated with brine.

Results and discussions

To investigate into the variation of the reflection amplitude of a plane seismic wave incident on a horizontal interface separating a HTI medium from an isotropic overburden, we consider the following properties of the two layers (Table 1). Figure 3 shows the variation of reflection coefficient with angle of incidence and with azimuth. It can be observed that for near normal incidence (up to around 10 degree), the underlying medium behaves like an isotropic medium showing practically no variation of reflection coefficient with variation of azimuth. However, for moderate and large angles of incidence, the reflection coefficient shows a sinusoidal variation with reflections showing peaks (maximum or minimum) along and perpendicular to the orientation of the fractures. Thus, orientation of the fractures can be determined from the analysis of AVOA data.

Figures (4a) and (4b) respectively show the variation of reflection coefficient with incidence angle from a fractured

Table 1 : Physical properties of an interface separating an isotropic medium from a fractured medium.

Medium	Density, gm/cc	α, km/s	β km/s	ΔN	ΔT
Isotropic	2.21	2.17	1.20	- -	- -
Vertically Fractured	2.00	2.00	1.00	0.05	0.10

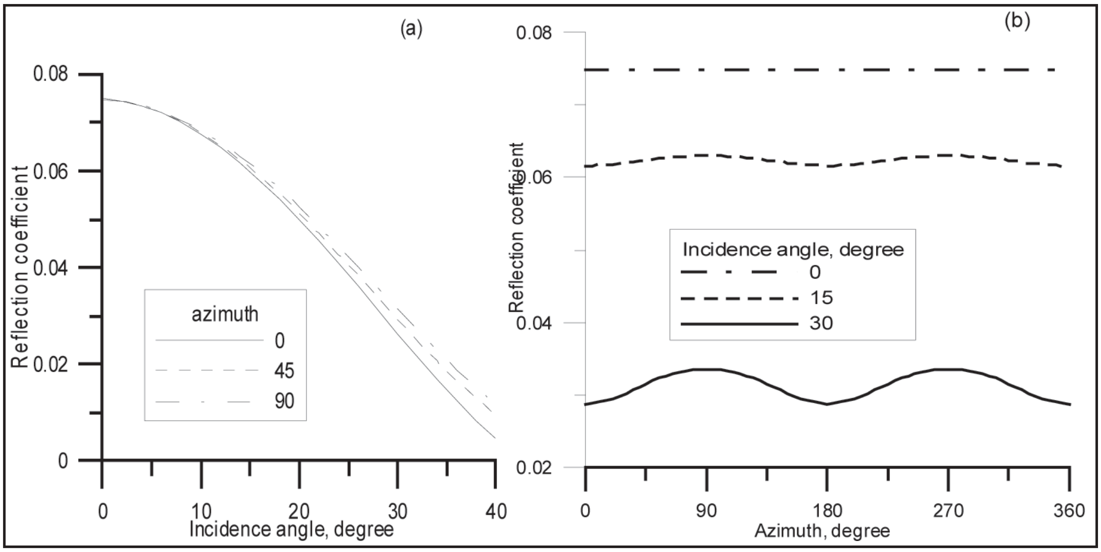
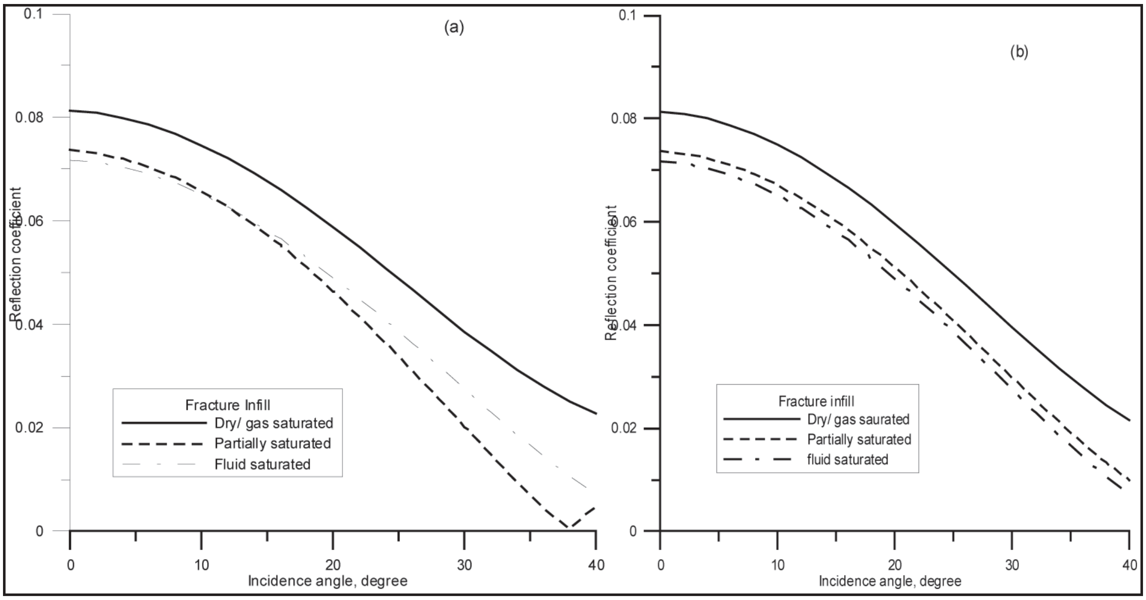


Fig. 3: Variation of reflection coefficients with (a) angles of incidence and (b) azimuth



medium with different fluid saturation as observed in along a seismic line perpendicular and parallel to the orientation of the fractures. It can be seen that dry and fluid saturated fractures have significantly different reflection coefficients. A combined analysis of AVO measured along and perpendicular to the fracture orientation or alternatively AVOA analysis, if the observed data contains adequate azimuth coverage, allows estimation of the nature of fluid infill in the fractures. Knowledge of the elastic properties of the background medium provides good constraints in estimating excess fracture compliances and hence in quantifying the fluid saturation. To give an idea of reliability of such results, we present the our earlier previous results (Shaw and Sen, 2005) in Table 2. It can be seen from Table-2 that the inversion of P- wave AVOA data yields satisfactory estimates of fracture weakness.

NMO velocity inversion

Azimuthal variation of NMO velocity over a fractured reservoir will constrain the fracture parameters in an effective way. The fracture and the background parameters can be estimated using the NMO ellipses from horizontal reflector in the direction of fracture strike. In this section linearized expressions for the vertical velocities and NMO ellipse of P, S_1 and S_2 waves and for the slowness surface of P-waves have been presented (Grechka, Bakulin and Tsvankin,2003). The goal of employing approximations is to identify the seismic signatures needed to constrain different fracture weaknesses.

Table 2 : Inversion of P-wave AVOA data to estimate fracture weakness.

Nature of fracture infill	**Normal Excess Compliance**		**Shear Excess Compliance**	
	True	**Estimated**	**True**	**Estimated**
Dry or gas filled	0.15	0.158	0.10	0.100
Partially filled	0.03	0.048	0.10	0.107
Fluid filled	0.00	0.006	0.15	0.175

NMO ellipses

Azimuthally varying NMO velocity of any pure-mode reflected wave is described by (Grechka, Bakulin and Tsvankin,2003).

$$V_{nmo}^{-2}|_P(\phi)=\frac{1}{\alpha^2}+\frac{1}{\alpha^2}\left\{1-4g\sin^2\phi-4g^2\cos 2\phi\right\}.$$

$$\Delta N+\frac{1}{\alpha^2}\left\{4g\cos^2\phi\right\}\Delta T \text{ (for P-wave)}$$

$$V_{nmo}^{-2}|_{S1}(\phi)=\frac{1}{\beta^2}+\frac{1}{\beta^2}\cos^2\phi\Delta T \text{ (for S1 wave)}$$

$$V_{nmo}^{-2}|_{S2}(\phi)=\frac{1}{\beta^2}+\frac{1}{\beta^2}\left\{4g\cos^2\phi\right\}\Delta N+\frac{1}{\beta^2}\left\{4\sin^2\phi-3\right\}\Delta T$$

(for S2 wave)

The approximations for the vertical velocities and NMO ellipses of reflected wave help to evaluate the

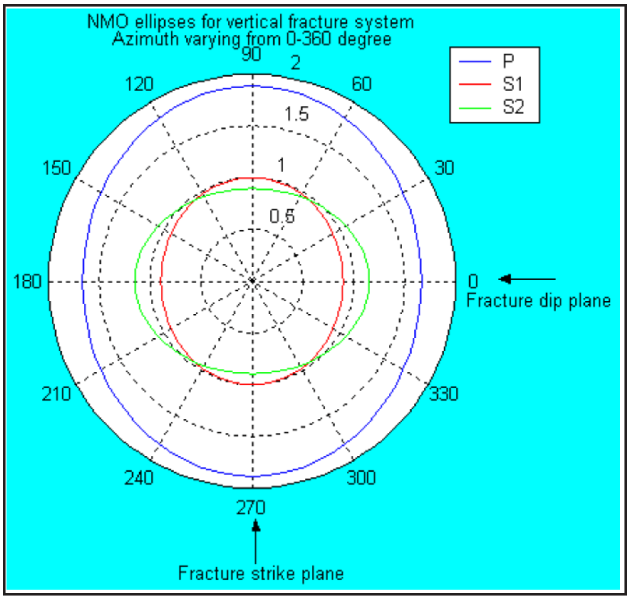


Fig 5: NMO ellipses over a homogeneous horizontal layer containing a vertical fracture system.

feasibility of estimating the fracture weaknesses and the background parameters from seismic data.

Effects of fluid infill on the AVOA over obliquely dipping fractures

Most of the investigations on seismic wave propagation in fractured media consider vertical or steeply dipping fractures. Angerer (2002) reported the existence of obliquely dipping fractures in hydrocarbon reservoirs. Grechka and Tsvankin (2004) developed a method for quantitative characterization of obliquely dipping fractures. In an earlier work (Shaw and Sen, 2004), they used Born approximation to write an approximation of linearized PP-reflection coefficients over a transversely isotropic medium with tilted axis of symmetry (TTI) in terms of weak anisotropic parameters.

Whereas variation of seismic amplitudes with azimuth provides information on the strike of the fractures, the dip of the fractures can not be estimated easily. The presence of obliquely dipping fractures can be inferred from the following observations on single/ multi component seismic data.

(i) asymmetry of travel times in mode converted PS-waves (Angerer et al, 2002) reflected from a horizontal reflector.

(ii) a significant horizontal component of P-wave as well as vertical component of slow S-wave, for near vertical incidence (Grechka and Tsvankin, 2004).

We used the reflectivity method to compute the AVO responses over obliquely dipping fractures along and perpendicular to the strike directions for dry as well as fluid filled fractures.

Results

First, we investigated the variation of reflection coefficient with azimuth. For this, we selected an incidence angle 30° and computed the PP-reflection coefficient for azimuth varying between 0° and 180° at an interval of 10° for both dry and fluid saturated fractures with dip of the symmetry axis 30° , 45° and 60°. The results (Fig. 7) show that AVOA for obliquely dipping fractures exhibit behavior similar to that over vertical fractures (Ruger, 1998). Thus, the strike direction of obliquely fractures can be determined from the azimuthal variation of reflection amplitudes, to an ambiguity of 90°.

Further, we selected a set of parallel fractures with the dip of the symmetry axis as 0° (vertical fractures), 30°, 45°, 60° and 90° (horizontal fractures, VTI medium). For each dip of the fractures, we computed the exact PP-reflection coefficient along and perpendicular to the fracture strike for incidence angles varying between 0° and 40° at an interval of 2°. Figures 6. (a) and 6(b) shows the variation of PP-reflection coefficient over obliquely dipping dry and fluid saturated fractures respectively. A comparison of these figures show that, irrespective of the dip of the fractures, (i) the PP-reflection coefficients over dry fractures are greater than those of fluid filled fractures (ii) appreciable azimuthal variation could be observed for angles of incidence exceeding 15°. Further, the variation of AVOA is best reflected for dips near 45° for dry fractures and for vertical fractures if saturated with fluid.

Following Grechka and Tsvankin's (2004) representation of elastic stiffness coefficients over obliquely dipping fractured medium in terms of normal and tangential weaknesses of the fracture system, we investigated into the effects of fluid infill on the AVOA over obliquely dipping fractures. we observed that the pattern of variation of AVOA over obliquely dipping fracture is similar to that over vertical fractures, so that the strike of the obliquely dipping fractures can be determined from the peak/ troughs of (co)sinusoidal variation of the amplitudes, to an ambiguity of 90°. For

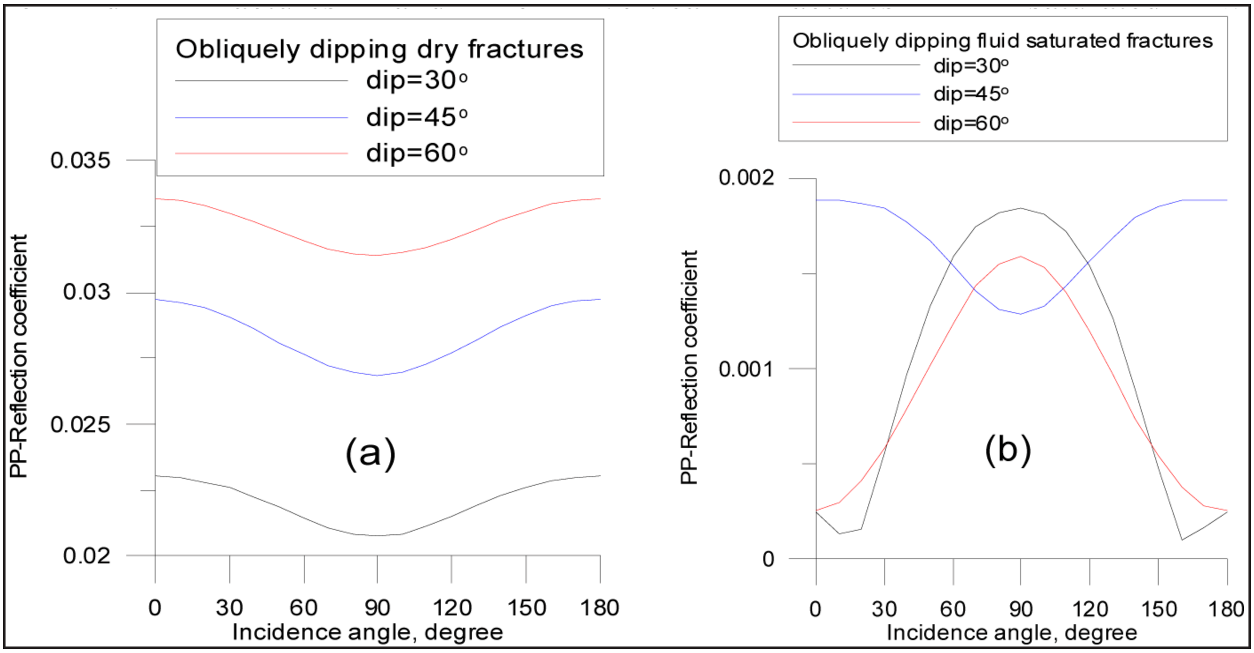


Fig.6: Amplitude variation with azimuth for different dips of fractures for (a) dry fractures and (b) fluid saturated fractures. The angle of incidence has been fixed at 30 degree.

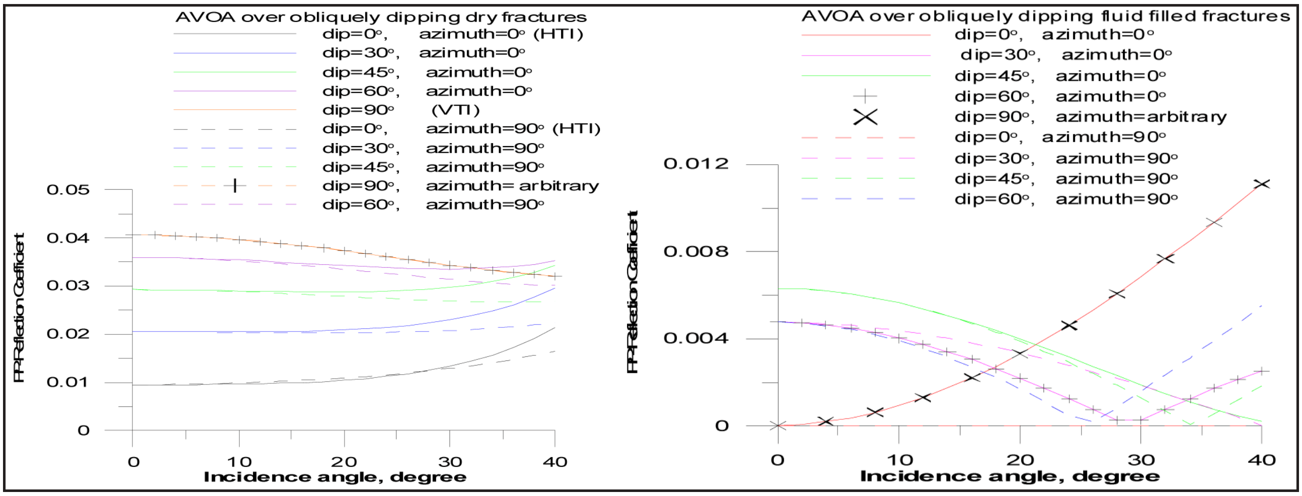


Fig. 7: AVO over obliquely dipping dry fractures along and perpendicular to fracture strike

typical sedimentary environments with the S-wave velocity half that of P-wave, the AVOA over obliquely dipping dry fractures are stronger than those over fluid saturated fractures, irrespective of the dip of the fractures. The fluid infill has an appreciable effect on the AVOA of dipping fractures.

Avo over visco-elastic media

The most severe effect of an anisotropic fractured overburden is attenuation due to fluid flow between the cracks and pores. A correct amplitude analysis at the target will not be possible unless the effect of attenuation is removed. Hudson, Liu and Crampin (1996) and Pointer have given theories describing anisotropic attenuation due to fluid flow in cracked porous rock as a function of structural parameters, fluid properties and frequency. We first introducing quality factors (Q_P and Q_S) and use complex velocities to specify intrinsic attenuation effects for elastic wave in visco-elastic media.Complex velocities are introduced by performing the following transformation:

$$\alpha \rightarrow \alpha\left(1-\frac{i}{2Q_P}\right), \beta \rightarrow \beta\left(1-\frac{i}{2Q_S}\right)$$

$$\alpha_0 \rightarrow \alpha_0\left(1-\frac{i}{2{Q_P}^0}\right), \ \beta_0 \rightarrow \beta_0\left(1-\frac{i}{2{Q_S}^0}\right)$$

$$R_{PP}^{VISCO} = R_{PP}^{Q_\infty} + R_{PP}^{Q}$$

Where

$$R_{PP}^{Q_\infty} = \frac{1}{2}\frac{\Delta Z}{\bar{Z}} + \frac{1}{2}\left[\frac{\Delta\alpha}{\bar{\alpha}} - 4\left(\frac{\bar{\beta}}{\bar{\alpha}}\right)^2 \frac{\Delta G}{\bar{G}}\right]\sin^2 i + \frac{1}{2}\frac{\Delta\alpha}{\bar{\alpha}}\sin^2 i \tan^2 i \ ,$$

is Reflection coefficient for lossless isotropic medium i.e. for Q=∞ and

$$R_{PP}^{Q} = \frac{j}{2\bar{Q}_P}\left[\left\{\frac{1}{2}\frac{\Delta Q_P}{\bar{Q}_P}\right\} + \left\{\frac{1}{2}\frac{\Delta Q_P}{\bar{Q}_P} - 8\left(\frac{\bar{\beta}}{\bar{\alpha}}\right)^2 \frac{\bar{Q}_P}{\bar{Q}_S}\frac{\Delta Q_S}{\bar{Q}_S}\right\}\right.$$

$$\left.\sin^2 i + \left\{\frac{1}{2}\frac{\Delta Q_P}{\bar{Q}_P}\right\}\sin^2 i \tan^2 i\right]$$

is the contribution to reflection coefficient from the pores acting as scatterer in a visco-elastic media.

Conclusions

We represented the fractures in a medium as a volume distribution of scatterers of seismic waves embedded in an isotropic medium. Superposition of singly scattered waves at a receiver far from the source as well as the scatterers linearizes the complex wavefield. Thus, investigation of the amplitude variation with offset over fractured medium becomes easy. We linearized the expression for P-wave AVOA over a vertically fractured medium in terms of the excess fracture compliances instead of transforming these into Thomsen parameters. We further investigated into the effects of fluid infill on the AVOA response. Our results show that besides yielding information about the fracture orientation, P-wave AVOA data contain valuable information regarding the saturation of the fracture in fill.

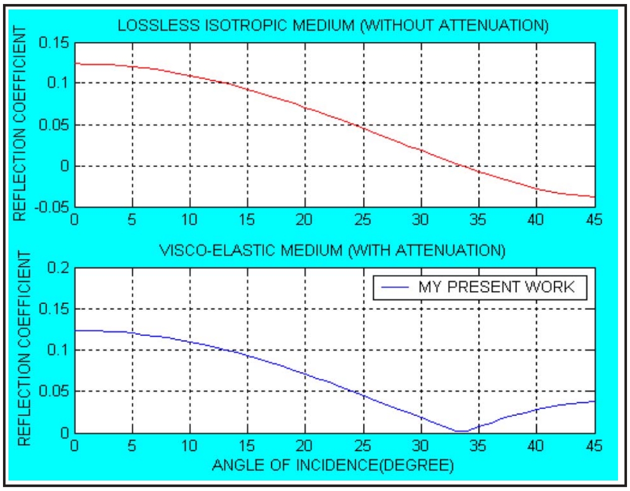


Fig. 8: Attenuation effect in visco-elastic media

The fractures in a medium can be represented as a volume distribution of scatterers of seismic waves embedded in an isotropic medium. Superposition of singly scattered waves at a receiver far from the source as well as the scatterers linearizes the complex wavefield. Thus, investigation of the amplitude variation with offset over fractured medium becomes easy. The expression for P-wave AVOA over a vertically fractured medium has been linearized in terms of the excess fracture compliances instead of transforming these into Thomsen parameters. The elastic properties of a set of vertical parallel fractures embedded in an otherwise isotropic medium are represented by general slips to derive the sensitivity of PP- and PS-wave reflection amplitudes on fracture weaknesses. From analytic results it is found that PP-AVOA does not contain information on Δ_{NV} and Δ_{VH} on the other hand, PS AVOA is sensitive to these parameters. The interfacial compliances describing the rock discontinuities may be regarded as macroscopic rock parameters and can be estimated from seismic data. Normal to shear compliance ratio is directly related to fluid saturation. Further the effects of fluid infill on the NMO velocity and AVOA response have been investigated. Results show that besides yielding information about the fracture orientation, NMO velocities and P-wave AVOA data contain valuable information regarding the saturation of the fracture in fill. Results from inversion of synthetic data show that the information on the fluid saturation can be inferred directly without mapping the fracture weaknesses to weak anisotropy parameters. Sinusoidal variation of seismic reflection amplitude helps identification of fracture orientation. Inversion of AVOA data for excess compliances yields reliable estimate of fluid saturation in natural fractures.

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